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Two Fluid Scenario for Axially Symmetric Dark Energy Cosmological Models in Brans-Dicke Theory of Gravitation

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Abstract. Spatially homogeneous axially symmetric cosmological models filled with barotropic fluid and dark energy are obtained in a scalar tensor theory of gravitation proposed by Brans and Dicke (1961). In these models one fluid is the radiation distribution which represents the cosmic microwave background and the other fluid is the perfect fluid representing the matter content of the universe. Also some important features of the models, thus obtained, have been discussed.

Keywords: Axially symmetric metric, Brans-Dicke theory, Two fluid, Dark energy, EoS parameter.

1 Introduction

Recently, there has been considerable interest in cosmological model with "Dark Energy" (DE) in general relativity because of the fact that our universe is currently undergoing an accelerated expansion which has been confirmed by host of observations, such as type I supernovae (SNeIa) ([1],[2],[3]), Sloan Digital Sky Survey (SDSS) [4], Wilkinson Microwave Anisotropy Probe (WMAP) ([5],[6],[7]). Based on these observations, cosmologists have accepted the idea of dark energy. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. Current studies to extract the properties of a dark energy component of the universe from observational data focus on the determination of its equation of state $\omega(t) = p / \rho$, which is not necessarily constant. The methods for restoration of the quantity $\omega(t)$ from expressional data have been developed [8], and an analysis of the experimental data has been conducted to determine this parameter as a function of cosmological time [9]. Recently the parameter $\omega(t)$ has been calculated with some reasoning which reduced to some simple parameterization of the dependences by some authors ([10], [11], [12], [13], [14],[15]). These observations provide us a clear outline of the universe: it is flat and full of un-damped form of energy density pervading the Universe. The un-damped energy called "Dark Energy" (DE) with negative pressure attributes to about 74 percent of the total energy density. The remaining 26 percent of the energy density consists of matter including about 22 percent dark matter density and about 4 percent baryon matter density. So, understanding the nature of DE is one of the most challenging problems in modern astrophysics and cosmology. Recent cosmological observations contradict the matter dominated universe with decelerating expansion indicating that our universe experiences accelerated expansion.

Jamil et al. [16] discussed thermodynamics of dark energy interacting with dark matter and radiation. [17] studied some FRW models of accelerating universe with dark energy. [18] discussed Bianchi type V, III and five dimensional dark energy models respectively in scalar-tensor theory of gravitation. [19],[20] obtained interacting two-fluid scenario for dark energy in anisotropic Bianchi type space-times. [21] discussed two-fluid scenario for dark energy model in a scalar tensor theory of gravitation. [22],[23],[24] and [25],[26],[27] have studied an interacting and non-interacting two-fluid scenario for dark energy models in FRW universe.

Brans-Dicke theory of gravitation is a natural extension of general relativity which introduces an additional scalar field φ besides the metric tensor g_{ij} and dimensionless coupling constant ω. The Brans-Dicke field equations for combined scalar and tensor field are given by

$$G_{ij} = -8\pi\varphi^{-1}T_{ij} - \omega\varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) - \varphi^{-1}(\varphi_{i;j} - g_{ij}\varphi_{;k}^{\;,k}) \tag{1}$$

and

$$\varphi_{;k}^{\ ,k} = 8\pi(3+2\omega)^{-1}T \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

Also, we have energy–conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

This equation is a consequence of the field equations (1) and (2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. [28] have obtained axially symmetric string cosmological models in Brans-Dicke theory of gravitation. [29] have discussed Bianchi type-II, VIII and IX magnetized cosmological models in Brans-Dicke theory of gravitation. [30],[31] have studied a higher-dimensional string cosmological model in a scalar-tensor theory of gravitation and Bianchi type-II, VIII and IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation. [32] have obtained LRS Bianchi type-I dark energy cosmological model in Brans-Dicke theory of gravitation. [33] have studied Kantowski-Sachs bulk viscous string cosmological model in Brans-Dicke theory of gravitation. Recently, [34],[35],[36] have discussed different aspects in this scalar tensor theory.

Inspired by the above investigations and discussion, in this paper, we study certain possible axially symmetric two fluids (barotropic fluid and dark energy) in Brans-Dicke theory of gravitation. Here we consider both interacting and non-interacting fluids.

2 Metric and Energy Momentum Tensor

We consider the Axially Symmetric metric [37], in the form

$$ds^2 = dt^2 - A^2(dx^2 + f^2(x)d\varphi^2) - B^2 dz^2 \tag{4}$$

where A, B are functions of 't' only and f is a function of the coordinate x only. The energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{5}$$

where ρ is the energy density, p is the pressure, u^i is the four-velocity of the fluid and

$$g_{ij}u^i u^j = 1 \tag{6}$$

In a co-moving coordinate system, we get

$$T_1^{\ 1} = T_2^{\ 2} = T_3^{\ 3} = -p,\ T_4^{\ 4} = \rho \text{ and } T_j^{\ i} = 0 \text{ for } i \neq j \tag{7}$$

where the quantities ρ and p are functions of 't' only.

3 Solutions of Field Equations

The field equations for the metric (4) with the help of equations (5), (6) and (7) can be written as

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} + \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 8\pi\varphi^{-1}p_{tot} \tag{8}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{2\dot{A}}{A}\right) = -8\pi\varphi^{-1}p_{tot} \tag{9}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + \frac{2\dot{A}\dot{B}}{AB} - \frac{\omega\dot{\varphi}^2}{2\varphi^2} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 8\pi\varphi^{-1}\rho_{tot} \tag{10}$$

$$\ddot{\varphi}+\dot{\varphi}\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=8\pi(3+2\omega)^{-1}(\rho_{tot}-3p_{tot}) \tag{11}$$

$$\dot{\rho}+(\rho+p)\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=0 \tag{12}$$

where $\rho_{tot}=\rho_m+\rho_{de}$ and $p_{tot}=p_m+p_{de}$. Here ρ_m and p_m are energy density and pressure of barotropic fluid and ρ_{de} and p_{de} are energy density and pressure of dark fluid respectively. The equations of state (EoS) parameters of barotropic fluid and dark fluid are given by $\omega_m=\frac{p_m}{\rho_m}$ and $\omega_{de}=\frac{p_{de}}{\rho_{de}}$ respectively.

3.1 Non-Interacting Two-Fluid Model

First we consider that two fluids do not interact with each other. Hence the general form of conservation equation (12) leads us to write the conservation equations for the dark energy and barotropic fluid separately as

$$\dot{\rho}_{de}+\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)(\rho_{de}+p_{de})=0 \tag{13}$$

$$\dot{\rho}_{m}+\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)(\rho_{m}+p_{m})=0. \tag{14}$$

The EoS parameter of barotropic fluid ω_m is constant ([38],[39]), that is

$$\omega_m=\frac{p_m}{\rho_m}=const., \tag{15}$$

while ω_{de} has been allowed to be a function of time since the current cosmological data from SNIa, CMB and large scale structures mildly favor dynamically evolving dark energy crossing the phantom divide line (PDL).

In order to solve highly non-linear field equations (8)-(12), we assume that the shear scalar (σ^2) is proportional to expansion scalar (θ). This condition leads to

$$A=B^n \tag{16}$$

where A and B are the metric potentials and n is positive constant.

From equations (8), (9) and (16), we get

$$\left(n-1\right)\frac{\ddot{B}}{B}+2n\left(n-1\right)\frac{\dot{B}^2}{B^2}-\frac{\dot{B}\dot{\phi}}{B\phi}=0 \tag{17}$$

From equation (17), we get

$$B=\left(k_4\right)^{\frac{1}{k_3}}\left(at+b\right)^{\frac{1}{k_2k_3}} \tag{18}$$

$$B=\left(k_4\right)^{\frac{1}{k_3}}\left(at+b\right)^{\frac{1}{k_2k_3}} \tag{19}$$

From equations (16) & (19), we get

$$A=\left(k_4\right)^{\frac{n}{k_3}}\left(at+b\right)^{\frac{n}{k_2k_3}} \tag{20}$$

where $k_2=\frac{n-1}{r+n-1}$, $k_3=2n+1$ and $k_4=ak_1k_2k_3$.

The metric (4) can be written as

$$ds^2=dt^2-\left(k_4\right)^{\frac{2n}{k_3}}\left(at+b\right)^{\frac{2n}{k_2k_3}}\left(dx^2+f^2(x)d\varphi^2\right)-\left(k_4\right)^{\frac{2}{k_3}}\left(at+b\right)^{\frac{2}{k_2k_3}}dz^2 \tag{21}$$

From equations (14), (19) and (20), we get

$$\rho_m = \rho_0 \left(k_4 \left(at+b\right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m)}. \tag{22}$$

where ρ_0 is an integration constant.

From equations (10), (18), (19), (20) and (22), we get the energy density

$$8\pi\rho_{de} = a^2 \left(\frac{2n^2+4n+2nrk_2k_3+2rk_2k_3-\omega r^2k_2^2k_3^2}{2\left(k_2k_3\right)^2\left(at+b\right)^{(2-r)}} \right) - 8\pi\rho_0 \left(k_4 \left(at+b\right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m)} \tag{23}$$

and the pressure

$$\begin{aligned} 8\pi p_{de} = -a^2 &\left(\frac{6n^2+4n-4nk_2-4nk_3+4n\ rk_2k_3+2r(r-1)k_2^2k_3^2+\omega r^2k_2^2k_3^2}{2\left(k_2k_3\right)^2\left(at+b\right)^{(2-r)}} \right) \\ &-8\pi\rho_0\omega_m \left(k_4 \left(at+b\right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m)} \end{aligned} \tag{24}$$

From equations (23) and (24), we get the EoS parameter

$$\omega_{de} = -\frac{\begin{array}{c} a^2\left(6n^2+4n-4nk_2-4nk_3+4nrk_2k_3+2r(r-1)k_2^2k_3^2+\omega\ r^2k_2^2k_3^2\right)+ \\ 16\,\pi\rho_0\omega_m \left(k_4 \left(at+b\right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m)} \left(k_2k_3\right)^2\left(at+b\right)^{(2-r)} \end{array}}{\begin{array}{c} a^2\left(2n^2+4n+2nrk_2k_3+2rk_2k_3-\omega\ r^2k_2^2k_3^2\right)- \\ 16\,\pi\rho_0 \left(k_4 \left(at+b\right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m)} \left(k_2k_3\right)^2\left(at+b\right)^{(2-r)} \end{array}}. \tag{25}$$

Thus the metric (21) together with (22) - (25) constitutes a non-interacting two-fluid scenario for axially symmetric dark energy cosmological models in Brans-Dicke theory of gravitation.

3.2 Interacting Two Fluid Model

In this section, we consider the interaction between dark and barotropic fluids. For this purpose we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_{de} + \left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)(\rho_{de}+p_{de}) = -Q \tag{26}$$

$$\dot{\rho}_m + \left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)(\rho_m+p_m) = Q \tag{27}$$

the quantity Q expresses the interaction between the dark energy components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q>0$ which ensures that the second law of thermodynamics is fulfilled. Here we emphasize that the continuity equations (26) and (27) imply that the interaction term (Q) should be proportional to a quantity with units of inverse of time i.e. $Q \propto \frac{1}{t}$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following [40] and [41], we consider

$$Q = 3H\sigma\rho_m \tag{28}$$

where σ is a coupling constant.

From equations (27) & (28), we get

$$\rho_m = \rho_0(A^2B)^{-(1+\omega_m-\sigma)} \tag{29}$$

where ρ_0 is an integration constant.

From equations (29), (19) and (20), we get

$$\rho_m = \rho_0 \left(k_4 \left(at + b \right)^{\frac{1}{k_2}} \right)^{-\frac{(2n+1)(1+\omega_m - \sigma)}{k_3}} \tag{30}$$

By using (30) in (10) & (9), we obtain density (ρ_{de}) and pressure (p_{de}) of dark energy as

$$8\pi\rho_{de} = a^2 \left(\frac{2n^2 + 4n + 2nrk_2k_3 + 2rk_2k_3 - \omega r^2 k_2^2 k_3^2}{2\left(k_2k_3\right)^2 \left(at+b\right)^{(2-r)}} \right) - 8\pi\rho_0 \left(k_4 \left(at + b \right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m - \sigma)} \tag{31}$$

$$\begin{aligned} 8\pi\, p_{de} = & -a^2 \left(\frac{6n^2 + 4n - 4nk_2 - 4nk_3 + 4nrk_2k_3 + 2r(r-1)k_2^2k_3^2 + \omega r^2 k_2^2 k_3^2}{2\left(k_2k_3\right)^2\left(at+b\right)^{(2-r)}} \right) \\ & + 8\pi \rho_0 \omega_m \left(k_4 \left(at+b \right)^{\frac{1}{k_2}} \right)^{-(1+\omega_m - \sigma)}. \end{aligned} \tag{32}$$

From equations (31) and (32), we get the EoS parameter

$$\omega_{de} = -\frac{\begin{array}{c} a^2\left(6n^2 + 4n - 4nk_2 - 4nk_3 + 4nrk_2k_3 + 2r(r-1)k_2^2k_3^2 + \omega r^2k_2^2k_3^2\right) + \\ 8\pi\rho_0 \left(k_4\left(at+b\right)^{\frac{1}{k_2}}\right)^{-(1+\omega_m-\sigma)} \left(k_2k_3\right)^2\left(at+b\right)^{(2-r)} \end{array}}{\begin{array}{c} a^2\left(2n^2 + 4n + 2nrk_2k_3 + 2rk_2k_3 - \omega r^2k_2^2k_3^2\right) + \\ 16\pi\rho_0\omega_m \left(k_4\left(at+b\right)^{\frac{1}{k_2}}\right)^{-(1+\omega_m-\sigma)} \left(k_2k_3\right)^2\left(at+b\right)^{(2-r)} \end{array}}. \tag{33}$$

Thus the metric (21) together with (29) - (33) constitutes a interacting two-fluid scenario for axially symmetric dark energy cosmological models in Brans-Dicke theory of gravitation.

4 Some Important Properties of the Models

The spatial volume (V), average scale factor (a) and expansion scalar (θ) are given by

$$V = A^2B = k_4\left(at+b\right)^{\frac{1}{k_2}} \tag{34}$$

$$\theta = 3H = \frac{a\left(2n+1\right)}{k_2k_3(at+b)} \tag{35}$$

$$\theta = 3H = \frac{a\left(2n+1\right)}{k_2k_3(at+b)} \tag{36}$$

The directional Hubble parameters H_1 , H_2 , and H_3 along the directions x , y and z are respectively given by

$$H_1 = H_2 = \frac{\dot{A}}{A} = \frac{na}{k_2k_3\left(at+b\right)} \tag{37}$$

$$H_3 = \frac{\dot{B}}{B} = \frac{a}{k_2k_3\left(at+b\right)} \tag{38}$$

whereas the generalized Hubble parameter is given by

$$H = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{a\left(2n+1\right)}{3k_2k_3(at+b)}. \tag{39}$$

The shear scalar (σ^2), anisotropic parameter (A_m) and deceleration parameter (q) are given by

$$\sigma^2 = \frac{7}{18}\theta^2 = \frac{7}{18}\frac{a^2\left(2n+1\right)^2}{k^2{}_2k^2{}_3(at+b)^2} \tag{40}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2\left(n-1\right)^2}{\left(2n+1\right)^2} \tag{41}$$

$$q = -\frac{3\dot{\theta}}{\theta^2} - 1 = -\frac{r - 2\left(n-1\right)}{r + \left(n-1\right)} \tag{42}$$

Look-back time-red shift: The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at red shift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1 + z = \frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (35), we get

$$1 + z = \frac{a_0}{a} = \left(\frac{aT_0 + b}{aT + b}\right)^{\frac{1}{3k_2}} \tag{43}$$

This equation can also be expressed as

$$H_0\Delta T = 1 - (1+z)^2 \tag{44}$$

where H_0 is the Hubble's constant.

Luminosity distance:

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and, is given by

$$d_L = r_1(1+z)a_0 \tag{45}$$

where r_1 is the radial coordinate distance of the object at light emission and, is given by

$$r_1 = \int_T^{T_0}\frac{1}{a}\,\mathrm{d}T = \frac{3k_2k_4^{\frac{-1}{3}}}{a\left(3k_2 - 1\right)}\left(aT_0 + b\right)^{\frac{\left(3k_2-1\right)}{3k_2}}\left[1 - \left(1+z\right)^{\left(1-3k_2\right)}\right] \tag{46}$$

From equations (27) and (28), we get the luminosity distance

$$d_L = \frac{3k_2k_4^{\frac{-1}{3}}}{a\left(3k_2 - 1\right)}a_0\left(1+z\right)\left(aT_0 + b\right)^{\frac{\left(3k_2-1\right)}{3k_2}}\left[1 - \left(1+z\right)^{\left(1-3k_2\right)}\right] \tag{47}$$

The distance modulus

$$D\left(Z\right) = 5\log d_L + 25 \tag{48}$$

From equations (29) and (30), we get

$$D(z) = 5\log\frac{3k_2k_4^{\frac{-1}{3}}}{a\left(3k_2 - 1\right)}a_0\left(1+z\right)\left(aT_0 + b\right)^{\frac{\left(3k_2-1\right)}{3k_2}}\left[1 - \left(1+z\right)^{\left(1-3k_2\right)}\right] + 25 \tag{49}$$

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5 Conclusions

In this paper we have presented axially symmetric two fluid cosmological models in Brans-Dicke (1961) scalar tensor theory of gravitation. The models have singularity at $t = \frac{-b}{a}$ and the volume increases with the increase of time at $t = \frac{-b}{a}$. The expansion scalar θ, shear scalar σ and the Hubble parameter decrease with the increase of time. From (41), one can observe that $A_m \neq 0$ which indicates that this model is always anisotropic. Thus the model presented here is expanding, non-rotating and accelerating in a standard way. Here the deceleration parameter appearing with the negative sign implies accelerating expansion of the universe, which is consistent with the present day observations. We have obtained expressions for look-back time ΔT, distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance. All the models presented here are anisotropic, non-rotating, shearing and also accelerating.

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