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An Analytical Theory with Respect to the Earth's Zonal Harmonic Term J_2 in Terms of Eccentric Anomaly for Short-Term Orbit Predictions

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Abstract. A new non-singular, analytical theory with respect to the Earth's zonal harmonic term J_2 has been developed for short-periodic motion, by analytically integrating the uniformly regular KS canonical equations of motion using generalized eccentric anomaly '*E*' as the independent variable. Only one of the eight equations needs to be integrated analytically to generate the state vector, as a result of symmetry in the equations of motion, and the computation for the other equations is by changing the initial conditions. The integrals are much simpler than earlier obtained in [20] in terms of the independent variable '*s*'. Numerical results indicate that the solution is reasonably accurate for a wide range of orbital parameters during a revolution. The error in computing the most important orbital parameter 'semi-major axis' which is the measure of energy is less than five percentage during a revolution. The analytical solution can have number of applications. It can be used for studying the short-term relative motion of two or more space objects. It can also be useful in collision avoidance studies of space objects. It can be used for onboard computation in the navigation and guidance packages, where the modeling of J_2 effect becomes necessary.

Keywords: Hamilton's equations of motion, uniformly regular KS canonical elements, Earth's oblateness J2, short-term orbit predictions, analytical integration.

1 Introduction

The main problem in the artificial satellite theory is the motion of the particle under the effect of Earth's oblateness, namely the second zonal harmonic J_2 in the gravitational potential field. Any Earth satellite mission requires the precise computation of the orbital motion under the influence of this dominating perturbation. The non-integrability dynamics of the J_2 problem [1] allow the avenue for analytical theories to be developed. In the past, several authors treated this problem to obtain closed form solution either by using averaging methods or by approximations. Using a canonical formulation, Sterne [2] investigated the problem of motion under the effect of an oblate spheroid and the canonical approach in terms of Delaunay variables was used by Brouwer [3]. A number of analytical theories for the motion of Earth's satellite under the effect of Earth's first few zonal harmonic terms are available in the literature [4-12]. The KS transformation regularizes the non-linear equations of motion and converts into linear differential equations of a harmonic oscillator. KS formulation was used by Engels and Junkins [13] and Jezewski [14] for short-term orbit predictions with J_2 effect.

The KS uniform regular canonical equations of motion [15] are a particular canonical form where all the ten elements are constant for unperturbed two-body problem and are applicable to elliptic, parabolic and hyperbolic orbital motion. Sharma and James Raj [16] numerically integrated these equations to obtain accurate orbit prediction under the effect of Earth's oblateness with zonal harmonic terms up to J_{36}. Analytical theory in terms of KS elements with J_2 [17], [18] and with J_3 and J_4 [19] was developed by Sharma for short-term orbit predictions. James Raj and Sharma [20] analytically integrated the uniformly regular KS canonical elements with Earth's zonal harmonics J_2, J_3 and J_4. The independent variable, fictitious time '*s*' given by $dt / ds = r$ with t and r being the physical time and radial distance, respectively, and used for analytical integration, resulted in complicated integrals. Because of the complexity of the integrals in evaluation for practical problems, the utility of the analytical solution was limited for operational purposes.

In the present paper, we have developed a new non-singular analytical solution with J_2 in close form in eccentricity 'e' by analytically integrating the uniformly regular KS canonical equations of motion, using the generalized eccentric anomaly 'E' as the independent variable. The integrals are found to be much simpler than obtained in [20]. The solution can have number of applications. It can be used for studying the short-term relative motion of two or more space objects and in collision avoidance studies of space objects. It can be also useful for onboard computation in the navigation and guidance packages, where the modeling of J_2 effect becomes necessary.

2 Equations of Motion

The KS uniformly regular canonical equations of motion for the state vector in terms of the independent variable's' are [15, 20]

$$\frac{\mathrm{d}\alpha_i}{\mathrm{d}s} = -\frac{\partial \overline{H}}{\partial \beta_i}, \frac{\mathrm{d}\beta_i}{\mathrm{d}s} = \frac{\partial \overline{H}}{\partial \alpha_i}, \text{ for } i = 1,2,3,4 \tag{1}$$

The relation between 's' and generalized eccentric anomaly 'E' is given by

$$E = 2\sqrt{\alpha_0}\, s \tag{2}$$

In view of (2), equations (1) in terms of E can be written as

$$\frac{\mathrm{d}\beta_i}{\mathrm{d}E} = \frac{\partial \overline{H}}{\partial \alpha_i}\left(\frac{\mathrm{d}s}{\mathrm{d}E}\right), \frac{\mathrm{d}\alpha_i}{\mathrm{d}E} = -\frac{\partial \overline{H}}{\partial \beta_i}\left(\frac{\mathrm{d}s}{\mathrm{d}E}\right), \tag{3}$$

where $\frac{\mathrm{d}s}{\mathrm{d}E} = \frac{1}{2\sqrt{\alpha_0}}$.

For perturbed potential, the Hamiltonian is

$$\overline{H} = \frac{1}{4}\left\{\sum_{k=1}^{4} u_k^2\left(\alpha_i \beta_i\right)\right\} V\left(\alpha_i \beta_i\right) - \frac{K^2}{4}. \tag{4}$$

When the perturbation due to Earth's oblateness J_2 is considered, then Eq (4) becomes

$$\overline{H} = \frac{1}{4}\left(rV - K^2\right),$$

with $V\left(\alpha_i, \beta_i\right) = \frac{K^2 R^2 J_2}{2\tilde{r}^3}\left[-1 + 3\frac{x_3^2}{r^2}\right]$.

We have

$$\begin{gathered} h = 2\alpha_0 = \left(\frac{K^2}{r}\right) - \left(\frac{\sqrt{\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)}}{2}\right) - V \\ \left(\mathbf{u}, \frac{\partial V}{\partial \mathbf{u}}\right) = -2\left(n+1\right)V \end{gathered}, \tag{5}$$

with

$$u_i = \left(\frac{\beta_i}{\sqrt{\alpha_0}}\right)\sin\left(\frac{E}{2}\right) - \alpha_i \cos\left(\frac{E}{2}\right), \ w_i = \left(\alpha_i\sqrt{\alpha_0}\right)\sin\left(\frac{E}{2}\right) + \beta_i \cos\left(\frac{E}{2}\right),$$

$$\alpha_i = \left(\frac{w_i}{\sqrt{\alpha_0}}\right)\sin\left(\frac{E}{2}\right) - u_i\cos\left(\frac{E}{2}\right),\ \ \beta_i = \left(u_i\sqrt{\alpha_0}\right)\sin\left(\frac{E}{2}\right) + w_i\cos\left(\frac{E}{2}\right),$$

$$\frac{\mathrm{d}h}{\mathrm{d}s} = 0,\ \ r = \frac{\mathrm{d}t}{\mathrm{d}s},\ \ \alpha_0 \text{ is constant.}$$

$$\left(x,y,z\right) = L\left(\mathbf{u}\right)\mathbf{u}\ \ ,\ \ \left(\dot{x},\dot{y},\dot{z}\right) = 2L\left(\mathbf{u}\right)w / r\ ,$$

$$L\left(\mathbf{u}\right) = \begin{pmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{pmatrix},$$

$$x = \left(x_1, x_2, x_3\right) = L\left(\mathbf{u}\right)\mathbf{u},$$

$$r = \sqrt{\left(x_1^2 + x_2^2 + x_3^2\right)} = u_1^2 + u_2^2 + u_3^2 + u_4^2,$$

where h, K^2, R, E, r, J_2 are total energy, gravitational constant, Earth's equatorial radius, generalized eccentric anomaly, radial distance and second zonal harmonic term of Earth, respectively.

2.1 Initial Conditions

As in [15], for $x_1 < 0$:

$$\begin{aligned} u_1^2 + u_4^2 &= \left(r + x_1\right)/2, \\ u_2 &= \left(x_2u_1 + x_3u_4\right)/\left(r + x_1\right),, \\ u_3 &= \left(x_3u_1 - x_2u_4\right)/\left(r + x_1\right). \end{aligned}$$

for $x_1 \geq 0$:

$$\begin{aligned} u_2^2 + u_3^2 &= \left(r - x_1\right)/2, \\ u_1 &= \left(x_2u_2 + x_3u_3\right)/\left(r - x_1\right), \\ u_4 &= \left(x_3u_2 - x_2u_3\right)/\left(r - x_1\right). \end{aligned}$$

Furthermore,

$$\begin{aligned} w_1 &= \left(u_1\dot{x}_1 + u_2\dot{x}_2 + u_3\dot{x}_3\right)/2, \\ w_2 &= \left(-u_2\dot{x}_1 + u_1\dot{x}_2 + u_4\dot{x}_3\right)/2, \\ w_3 &= \left(-u_3\dot{x}_1 - u_4\dot{x}_2 + u_1\dot{x}_3\right)/2, \\ w_4 &= \left(u_4\dot{x}_1 - u_3\dot{x}_2 + u_2\dot{x}_3\right)/2. \end{aligned}$$

3 Analytical Integration

The right hand side of the equations (3) can be written as

$$\frac{\partial\bar{H}}{\partial\alpha_i}\frac{\mathrm{d}s}{\mathrm{d}E} = \frac{K^2R^2J_2}{8\sqrt{\alpha_0}}\left(\frac{1}{r^3}\frac{\partial r}{\partial\alpha_i} + \frac{3x_3}{r^4}\frac{\partial x_3}{\partial\alpha_i} - \frac{6x_3^2}{r^5}\frac{\partial r}{\partial\alpha_i}\right), \tag{6}$$

$$\frac{\partial\bar{H}}{\partial\beta_i}\frac{\mathrm{d}s}{\mathrm{d}E} = \frac{K^2R^2J_2}{8\sqrt{\alpha_0}}\left(\frac{1}{r^3}\frac{\partial r}{\partial\beta_i} + \frac{3x_3}{r^4}\frac{\partial x_3}{\partial\beta_i} - \frac{6x_3^2}{r^5}\frac{\partial r}{\partial\beta_i}\right), \tag{7}$$

where x_3 and x_3^2 are given as [16]:

$$\begin{aligned} x_3 &= a_0 + a_1\cos E + a_2\sin E \\ x_3^2 &= b_0 + b_1\cos E + b_2\cos^2 E + b_3\sin E + b_4\sin E\cos E \end{aligned},$$

with

$$a_0 = \alpha_1\alpha_3 + \alpha_2\alpha_4 + \frac{1}{\alpha_0}\left(\beta_1\beta_3 + \beta_2\beta_4\right),$$

$$a_1 = \alpha_1\alpha_3 + \alpha_2\alpha_4 - \frac{1}{\alpha_0}\left(\beta_1\beta_3 + \beta_2\beta_4\right),$$

$$a_2 = \frac{-1}{\sqrt{\alpha_0}}\left(\alpha_1\beta_3 + \beta_1\alpha_3 + \alpha_2\beta_4 + \beta_2\alpha_4\right),$$

$$b_0 = a_0^{\,2} + a_2^{\,2},$$

$$b_1 = 2a_0a_1,$$

$$b_2 = a_1^2 - a_2^2,$$

$$b_3 = 2a_0a_{2,}$$

$$b_4 = 2a_1a_2.$$

Substituting the values of x_3 and x_3^2 into the equations (6), we get

$$\begin{aligned}\frac{d\alpha_i}{dE} &= \frac{K^2R^2J_2}{8\sqrt{\alpha_0}}[\frac{1}{r^3}\left\{q_0^{(i)} + q_1^{(i)}\cos E + q_2^{(i)}\sin E\right\} \\ &+ \frac{3}{r^4}\left\{g_0^{(k)} + g_1^{(k)}\cos E + g_2^{(k)}\cos^2 E + g_3^{(k)}\sin E + g_4^{(k)}\cos E\sin E\right\} \\ &- \frac{6}{r^5}\left\{\begin{array}{l} f_0^{(i)} + f_1^{(i)}\cos E + f_2^{(i)}\cos^2 E + f_3^{(i)}\cos^3 E + f_4^{(i)}\sin E \\ + f_5^{(i)}\cos E\sin E + f_6^{(i)}\cos^2 E\sin E \end{array}\right\}],\end{aligned} \tag{8}$$

with

$$q_0^{(i)} = \frac{\beta_i}{\alpha_0}, \qquad q_1^{(i)} = \frac{-\beta_i}{\alpha_0}, \qquad q_2^{(i)} = \frac{-\alpha_i}{\sqrt{\alpha_0}},$$

for α_i variation.

For β_i variation, we have

$$q_0^{(i)} = \alpha_i, \qquad q_1^{(i)} = \alpha_i, \qquad q_2^{(i)} = \frac{-\beta_i}{\sqrt{\alpha_0}},$$

Also, $k = i + 2$, and

$$g_0^{(k)} = a_0q_0^{(k)} + a_2q_2^{(k)},$$

$$g_1^{(k)} = a_1q_0^{(k)} + a_0q_1^{(k)},$$

$$g_2^{(k)} = a_1q_1^{(k)} - a_2q_2^{(k)},$$

$$g_3^{(k)} = a_2q_0^{(k)} + a_0q_2^{(k)},$$

$$g_4^{(k)} = a_1q_2^{(k)} + a_2q_1^{(k)},$$

$$f_0^{(i)} = b_0q_0^{(i)} + b_3q_2^{(i)},$$

$$f_1^{(i)} = b_1q_0^{(i)} + b_0q_1^{(i)} + b_4q_2^{(i)},$$

$$f_2^{(i)} = b_2q_0^{(i)} + b_1q_1^{(i)} - b_3q_2^{(i)},$$

$$f_3^{(i)} = b_2q_1^{(i)} - b_4q_2^{(i)},$$

$$f_4^{(i)} = b_3q_0^{(i)} + b_0q_2^{(i)},$$

$$f_5^{(i)} = b_4q_0^{(i)} + b_3q_1^{(i)} + b_1q_2^{(i)},$$

$$f_6^{(i)} = b_4q_1^{(i)} + b_2q_2^{(i)}.$$

On substituting $r = a\left(1 - e\cos E\right)$ into equations (8) and integrating analytically, we get

$$\begin{aligned}\Delta\alpha_i = &\frac{K^2R^2J_2}{8a^3\sqrt{\alpha_0}}[q_0^{(i)}\wedge_3^{00} + q_1^{(i)}\wedge_3^{10} + q_2^{(i)}\wedge_3^{01}\\ &+\frac{3}{a}\left\{g_0^{(k)}\wedge_4^{00} + g_1^{(k)}\wedge_4^{10} + g_2^{(k)}\wedge_4^{20} + g_3^{(k)}\wedge_4^{01} + g_4^{(k)}\wedge_4^{11}\right\}\\ &-\frac{6}{a^2}\left\{f_0^{(i)}\wedge_5^{00} + f_1^{(i)}\wedge_5^{10} + f_2^{(i)}\wedge_5^{20} + f_3^{(i)}\wedge_5^{30} + f_4^{(i)}\wedge_5^{01} + f_5^{(i)}\wedge_5^{11} + f_6^{(i)}\wedge_5^{21}\right\}],\end{aligned} \tag{9}$$

where

$$\wedge_q^{ps} = \int \frac{\cos^p E \sin^s E}{\left(1 - e\cos E\right)^q}\,\mathrm{d}\,E,$$

$$\wedge_0^{00} = E,$$

$$\wedge_1^{00} = \frac{2}{\sqrt{\eta}}\tan^{-1}\left[\left(\frac{1+e}{1-e}\right)^{1/2}\tan\left(\frac{E}{2}\right)\right]$$

$$\wedge_n^{00} = \frac{1}{\left(n-1\right)\eta}\left[\frac{e?\quad E}{\varphi^{n-1}} + \left(2n-3\right)\wedge_{n-1}^{00} - \left(n-2\right)\wedge_{n-2}^{00}\right], \qquad n > 1$$

$$\wedge_n^{01} = -\frac{1}{\left(n-1\right)e\varphi^{n-1}}, \qquad n > 1$$

$$\wedge_n^{11} = \frac{1}{e}\left(\wedge_n^{01} - \wedge_{n-1}^{01}\right), \qquad n > 2$$

$$\wedge_5^{21} = \frac{1}{e^2}\left(\wedge_5^{01} - 2\wedge_4^{01} + \wedge_3^{01}\right),$$

$$\wedge_n^{m0} = \frac{1}{\left(-e\right)^m}\sum_{k=0}^{m}\binom{m}{k}\left(-1\right)^{m-k}\wedge_{n-k}^{00}$$

with $\varphi = \frac{r}{a}$, $\eta = 1 - e^2$.

4 Numerical Results

To compute the uniformly regular KS canonical elements with Earth's zonal harmonic term J_2 during a revolution, we have programmed equations (6) and (7) in double precision arithmetic. The numerical integration (NUM) is carried out using fourth-order Runge-Kutta-Gill method. The analytical solutions are obtained from equations (8). The canonical elements are converted to the state vectors and then to the orbital elements. Three test cases A, B, C with constant perigee altitude of 250 km and apogee altitudes of 250 (e=00379), 1000 (e=0.0573) and 10000 (e=0.4269) km at three different inclinations (5°, 30° and 85°) are chosen for detailed numerical studies. The initial conditions are given in Table 1. The bilinear relation

$$\alpha_4\beta_1 - \alpha_3\beta_2 + \alpha_2\beta_3 - \alpha_1\beta_4 = 0,$$

in terms of the canonical elements is utilized for finding the accuracy of numerical and analytical solutions. Figures 1, 2 and 3 are plotted for the three cases A, B, C with respect to numerical and analytical computations for the three inclinations 5, 30 and 85 degrees, respectively. Figures 1a, 1c and 1e provide the variation in osculating semi-major axis, eccentricity and inclination during a revolution for the three cases for 5 degree inclination. Similarly Figures 2a, 3a; 2c, 3c and 2e, 3e provide the variation in osculating semi-major axis, eccentricity and inclination during a revolution for the cases 2 and 3 for i=30 and 85 degrees, respectively. The maximum variations during a revolution in osculating semi-major axis are for case C (e=0.4269) and are around 18.6, 4.8 and 48 km for i=5, 30 and 85 degrees, respectively. The difference between the numerical and the analytical values computed in a single step (ANAL1) with respect to the independent variable E are provided in Figures 1b, 1d, 1f; 2b, 2d, 2f; 3b, 3d, 3f; in osculating semi-major axis, eccentricity and inclination during a revolution for the three cases for i=5, 30 and 85 degrees, respectively.

Table 1. Initial orbital parameters.

Parameters		Values		
		Case A	Case B	Case C
Perigee altitude	**(km)**	200	200	200
Apogee altitude	**(km)**	250	1000	10000
Semi-major axis	**(km)**	6603.14	6978.1	11478
Eccentricity		0.00379	0.0573	0.4269
Inclination	**(degree)**	5, 30, 85	5, 30, 85	5, 30, 85
Argument of perigee	**(degree)**	270	270	270
Mean Anomaly	**(degree)**	0	0	0

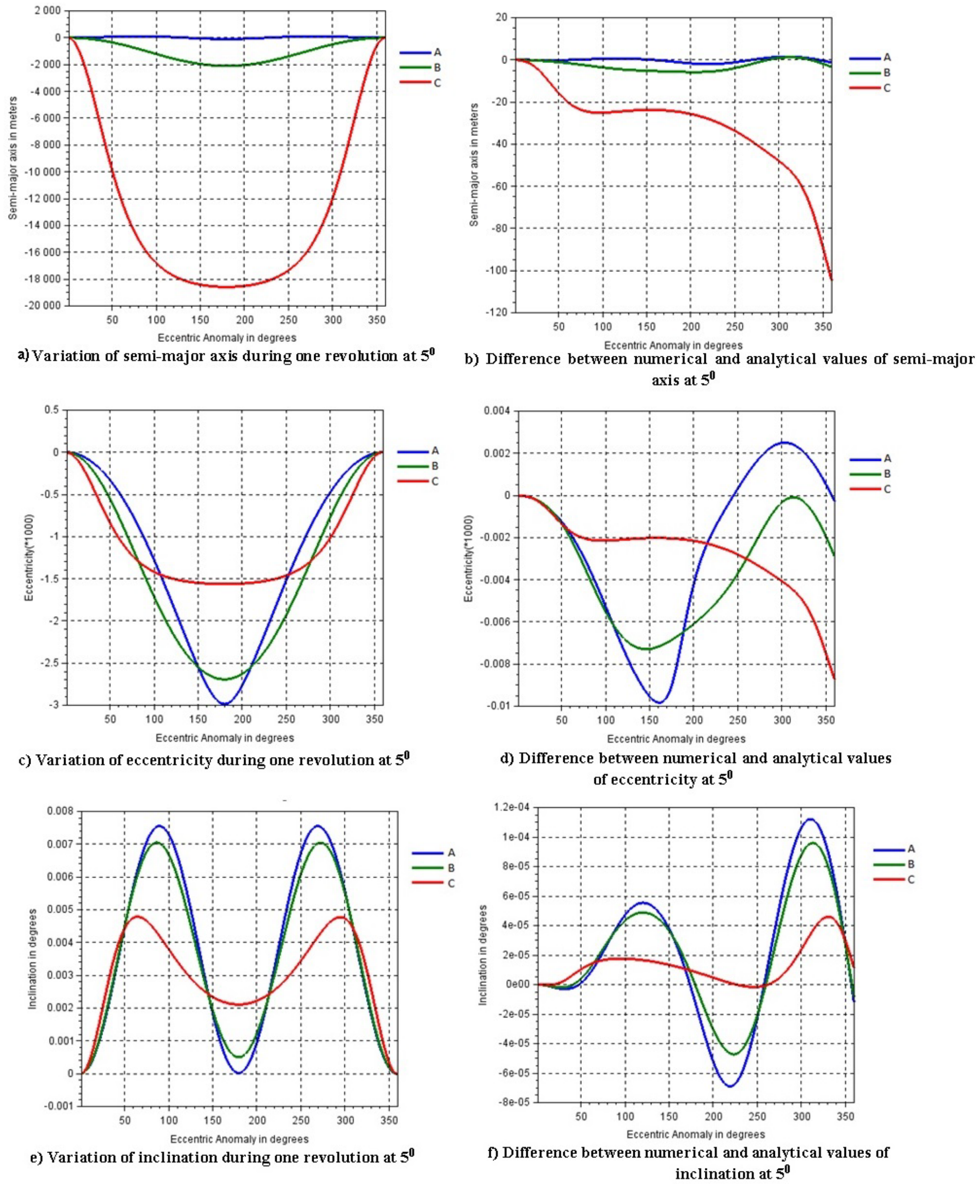
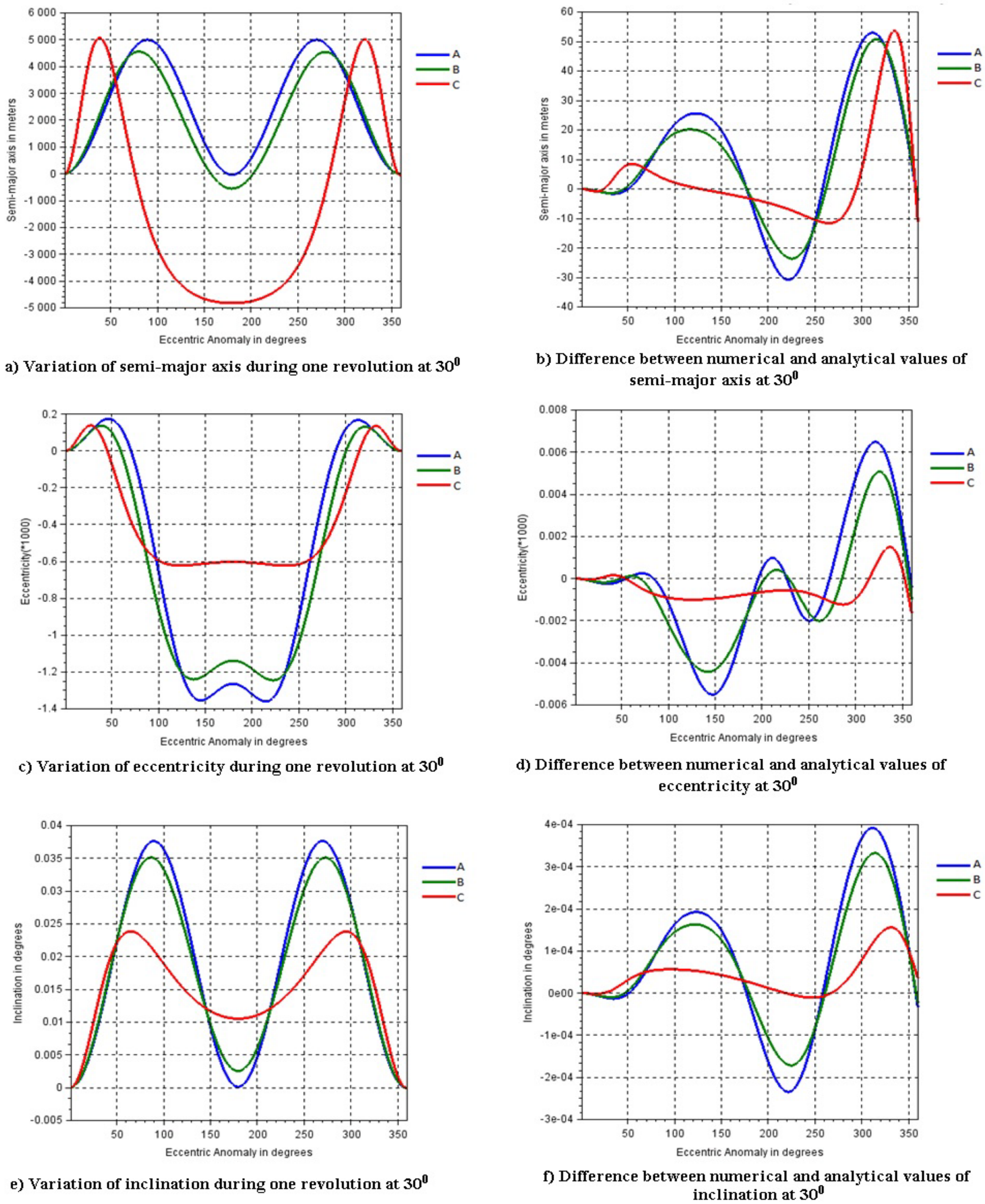


Figure 1. Analytical and numerical comparison at 5° inclination.

It is noticed from Fig. 1b (5 degrees inclination) that the difference between NUM and ANAL1 in osculating semi-major axis, increases with the increase in the analytical step size E for high eccentricity (0.4269) case C and is -107 metres at E=360 degrees. However, the difference is less than 3 metres for the cases A and B, whose initial eccentricities are small: 0.00379, 0.0573, respectively. The maximum difference between NUM and ANAL1 in semi-major axis is 54 metres at E=335 degrees and 181 metres at E=330 degrees for case C, for i=30 and 85 degrees. It is noted from Figs. 1-3 that the difference between NUM and ANAL1 up to half a revolution for three cases with the three inclinations is quite less.

Table 2. Variation in semi-major axis during half a revolution with J_2 for case B.

Parameter	**Method**	**ANAL steps (deg)**						
		10	**20**	**30**	**60**	**90**	**135**	**180**
a (m) Case B at 5º	ANAL1	-14.0251	-56.0713	-125.9331	-490.8953	-1018.6194	-1776.0763	-2078.024
	NUM-ANAL1 (single step)	-0.00073	-0.0115	-0.0575	-0.8397	-3.5202	-11.3816	-21.1429
	ANAL 2	-14.0254	-56.077	-125.9605	-491.2813	-1020.4034	-1784.0055	-2095.535
	NUM-ANAL2 (1 deg)	-0.0003	-0.0057	-0.0301	-0.4671	-1.7363	-3.4524	-3.6321
	ANAL 3	-14.0254	-56.0776	-125.9625	-491.2813	-1020.4215	-1783.9930	-2095.511
	NUM-ANAL3 (5 deg)	-0.0003	-0.0052	-0.0281	-0.4536	-1.7182	-3.4649	-3.6562
a (m) Case B at 30º	ANAL 1	185.4156	708.9041	1479.3575	3936.9155	4416.05673	1445.9347	-543.1845
	NUM-ANAL1 (single step)	-0.00233	-0.0376	-0.1931	-3.0411	-12.6871	-31.3681	-26.1483
	ANAL 2	185.4156	708.9041	1479.3575	3936.9155	4416.0567	1445.9347	-543.1845
	NUM-ANAL2 (1 deg)	0.00472	0.0821	0.4334	5.0712	7.7481	-0.1733	-1.3117
	ANAL 3	185.4178	708.9205	1479.4174	3937.2730	4416.4841	1446.6301	-542.4118
	NUM-ANAL3 (5 deg)	0.0025	0.0657	0.3735	4.7137	7.3206	-0.8687	-2.0844
a (m) Case B at 85º	ANAL 1	800.2371	3067.5175	6430.3174	17610.8634	21218.1106	11505.8182	4361.2771
	NUM-ANAL1 (single step)	-0.0587	-0.8812	-4.00873	-35.46	-59.3743	39.8935	107.7332
	ANAL2	800.3274	3068.7386	6434.5023	17606.7487	21187.8139	11566.601	4478.9959
	NUM-ANAL2 (1 deg)	-0.149	-2.1022	-8.1936	-31.3454	-29.0776	-20.8891	-9.9856
	ANAL3	800.2835	3068.4893	6434.0773	17608.2199	21190.1304	11570.7026	4483.0149
	NUM-ANAL3 (5 deg)	-0.1051	-1.8529	-7.7685	-32.8166	-31.3941	-24.9908	-14.0045



a) Variation of semi-major axis during one revolution at 30^0

b) Difference between numerical and analytical values of semi-major axis at 30^0

c) Variation of eccentricity during one revolution at 30^0

d) Difference between numerical and analytical values of eccentricity at 30^0

e) Variation of inclination during one revolution at 30^0

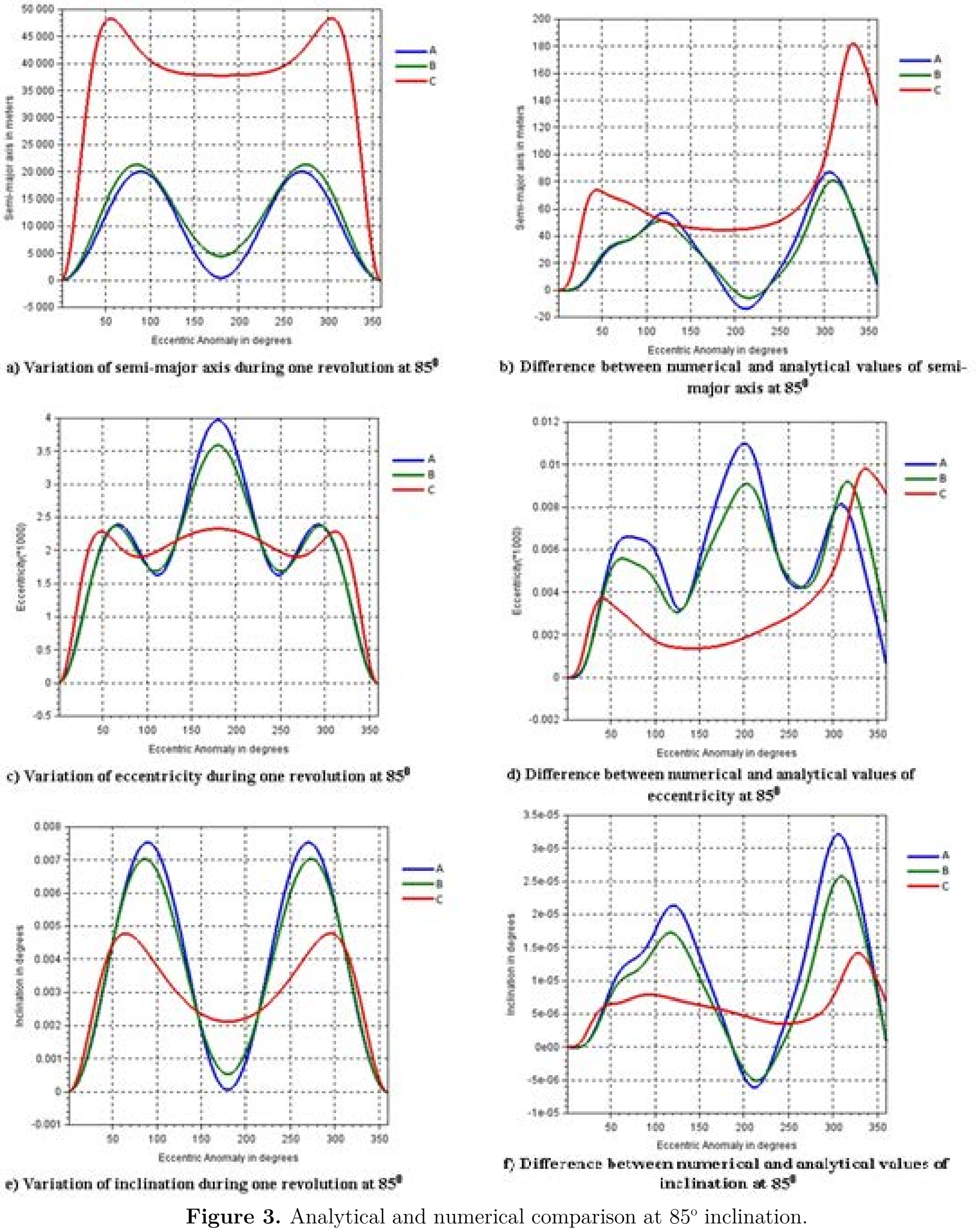
f) Difference between numerical and analytical values of inclination at 30^0

Figure 2. Analytical and numerical comparison at 30º inclination.

We conclude that the present analytical solution is suitable for computation of the state vectors in a single step up to half a revolution. In Table 2, we provide the difference between numerical and analytical values of semi-major axis for case B (e=0.0573) at 5, 30 and 85 degrees inclinations for half a revolution. ANAL1 is the difference between analytical integration in a single step and initial value of semi-major axis. ANAL2 is the analytical integration with one degree step size in E utilizing 180 integration steps minus the initial semi-major axis. ANAL3 is the analytical integration with a step size of 5 degree utilizing 36 integration steps minus the initial semi-major axis. The deviations during half a revolution with ANAL1, ANAL2 and ANAL3 and difference between NUM and ANAL1, ANAL2 and ANAL3 are provided at E=10, 20, 30, 60, 90, 135 and 180 degrees, respectively.

It may be noted that ANAL2 and ANAL3 improve the accuracy considerably during the half a revolution. It is interesting to note that the differences between NUM and ANAL1 with 30 degrees analytical step size, for the 3 inclinations of 5, 30 and 85 degrees are 0.057, 0.193 and

4.009 metres, respectively, over a variation of 125.9, 1479.4 and 6430.3 metres, respectively. The percentage error is only 0.045, 0.013 and 0.062, respectively, for the 3 eccentricities. It is also interesting to note that for orbit computation up to 60 degrees in E, single analytical step is sufficient. For higher values of E, a small analytical step size of 1 to 5 degrees provides more accurate orbits in the osculating state.



a) Variation of semi-major axis during one revolution at 85°

b) Difference between numerical and analytical values of semi-major axis at 85°

c) Variation of eccentricity during one revolution at 85°

d) Difference between numerical and analytical values of eccentricity at 85°

e) Variation of inclination during one revolution at 85°

f) Difference between numerical and analytical values of inclination at 85°

Figure 3. Analytical and numerical comparison at 85° inclination.

5 Conclusion

KS uniformly regular canonical equations of motion provide a very efficient and accurate analytical integration method for short-term orbit computation with Earth's oblateness J_2 for short-term motion during a revolution. Only one of the eight equations needs to be integrated analytically to generate the state vector as a result of symmetry in the KS uniformly regular canonical equations of motion. The integrals are much simpler to evaluate than obtained earlier. Numerical results indicate that the solution is reasonably accurate for a wide range of orbital parameters during half a revolution. The solution has number of applications. It can be used for studying the short-term relative motion of two or more space objects, collision avoidance studies of space objects and for onboard computation in the navigation and guidance packages, where the modeling of J_2 effect becomes necessary.

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References

1. A. Celleti, P. Negrini, “Non-integrability of the problem of motion around an oblate planet,” Celestial Mechanics, vol. 61, pp. 253-260, 1995.
2. T.E. Sterne, “The gravitational orbit of a satellite of an oblate planet,” Astronomical Journal, vol. 63, pp 28-40, 1958.
3. D. Brouwer, “Solution of the Problem of Artificial Satellite Theory without Drag,” Astronomical Journal, vol. 64, pp. 378-396, 1959.
4. D. King-Hele, “The effect of the Earth Oblateness on the Orbit of a Near Satellite,” Proc. R. Soc. London A vol. 247, pp. 49-72, 1958.
5. Y. Kozai, “The Motion of a Close Earth Satellite,” Astronomical Journal Astron. J, vol. 64, pp. 367-377, 1959.
6. B. Garfinkel, “The orbits of a satellite of an oblate planet,” Astronomical Journal, vol. 64, pp 353-, 1959.
7. A. Deprit, A. Rom, “The main problem of satellite theory for small eccentricities,” Celestial Mechanics, vol. 4, pp. 119-121, 1970.
8. K. Aksnes, “A Second-Order Artificial Satellite Theory Based on an Intermediate Orbit,” Astronomical Journal, vol. 75, pp 1066-1076, 1970.
9. H. Kinoshita, “Theory of rotation of the rigid Earth,” Celestial Mechanics, vol. 15, pp. 277-326, 1977.
10. A. Deprit, “The elimination of parallax in satellite theory,” Celestial Mechanics, vol. 24, pp.111-153, 1981.
11. R.H. Gooding, “On the Generation of Satellite Position (and Velocity) by a Mixed Analytical-Numerical Procedure,” Advances in Space Research, vol. 1, pp. 83-93, 1981.
12. R.H. Gooding, “Perturbations, untruncated in eccentricity, for an orbit in an axi-symmetric gravitational field”, Journal of Astronautical Science, vol. 39, pp. pp. 65-85, 1991.
13. R. C. Engels, J. L. Junkins, “The Gravity-Perturbed Lambert Problem: A KS Variation of Parameters Approach,” Celestial Mechanics, vol. 24, pp. 3-21, 1981.
14. D. J. Jezewski, “A noncanonical analytic solution to the J2 perturbed two-body problem,” Celestial Mechanics, vol. 30, pp. 343-361, 1983.
15. E. L. Stiefel, G. Scheifele, “Linear and Regular Celestial Mechanics,” Springer, Berlin, 1971.
16. R. K. Sharma and M. X. James Raj, “Long-term orbit computations with KS uniformly regular canonical elements with oblateness,” Earth Moon and Planets, vol. 42, pp. 163-178, 1988.
17. R. K. Sharma, “Analytical approach using KS elements to short-term orbit predictions including J2,” Celestial Mechanics, Celestial Mechanics and Dynamical Astronomy, vol. 46, pp. 321-333, 1989.
18. R. K. Sharma, “Analytical integration of KS element equations with J2 for short-term orbit predictions,” Planetary and Space Science, vol. 45, pp. 1481-1486, 1997.
19. R. K. Sharma, “Analytical short-term orbit predictions with J3 and J4 in terms of KS elements,” Celestial Mechanics and Dynamical Astronomy, vol. 56, pp. 503-521, 1993.
20. M. X. James Raj and R. K. Sharma, “Analytical short-term orbit prediction with J2, J3, J4 in terms of KS uniformly regular canonical elements,” Advances in Space Research, vol. 31, pp. 2019-2025, 2003.

