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Application of the Oscillation Symmetry to the Electromagnetic Interactions of Some Particles, Nuclei, and Atoms

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Abstract The oscillation symmetry is first applied to electromagnetic interactions of particles and nuclei. It is shown that the differences between successive masses plotted versus their mean values and the electromagnetic decay widths Γ_{ee} of $0^-(1^{--})$ $b\bar{b}$ and $c\bar{c}$ mesons, plotted versus their masses, agree with such symmetry. Then it is shown that the variation of the energy differences between different levels of several nuclei from ^{8}Be to ^{20}Ne, corresponding to given electric or magnetic transitions, display also oscillating behaviours. The electromagnetic widths of the electric and magnetic transitions between excited levels of these nuclei, plotted versus the corresponding differences between energies agree also with this property. The oscillating periods describe also an oscillation, the same for E1, M1, and E2 transitions. It is also the case for the multiplicative factor used β, and for ratios between these parameters.
The oscillation symmetry is then applied to atomic energy levels of several neutral atoms from hydrogen to phosphorus.
The data exhibit nice oscillations when plotted in the same way as before. The oscillations in various nuclear and atomic periods of different elements (A) exhibit the same shape and can be fitted by the same distribution.

Keywords: oscillation symmetry, masses, nuclear and electromagnetic interactions, particles, nuclei, atoms.

1 Introduction

The oscillation symmetry, was extended recently, from the classical world to the quantic one. Indeed here the masses are solution of the Schrödinger eq., containing kinetic and potential interactions, as are the oscillations of pendulums or springs. This was observed in particle and nuclei level masses [1, 2]. Then such oscillation property was also observed in different other data, suggesting that this property is rather general in nature. The oscillation property is observed, provided that the studied objects are submitted to opposite interactions and whatever the nature of the interactions involved. Opposite interactions act always when bodies result from smaller body combinations. Otherwise these bodies will either disintegrate or compress themselves and fuse up to loose their initial state as do plasmas for example. Such situation exists often in nature: for example nuclei made with nucleons, nucleons with quarks. This was already used to predict some data still unknown, like excited level spins.

The validity of this property was confirmed by different studies. These studies concern particles: mesons and baryons masses and widths, and excited states nuclei level masses and widths [1]. A short part of all these studies is given in [2].

There are also other situations in nature when opposite forces act on very separated bodies. These situations concern many astrophysical properties since the bodies are submitted to gravitationnal forces and centrifugal forces related to their kinetic energies. Planets are binded to their stars, galaxies to their black holes, and so on. Therefore the oscillation symmetry has been applied to the astrophysical properties [3]. It was observed that the model works and allows to predict some still unknown data like the mass of the seventh planet around TRAPPIST-1 star [3]. The model was also applied to attempt to predict some properties of the putative ninth and tenth new solar planets [4]. It allows to highlight remarkable relations between masses [5].

2 The Oscillation Symmetry

I start to classify the studied masses in increasing order. The possible oscillations in the mass spectra are studied using the following relation:

$$m_{(n+1)} - m_n = f[(m_{(n+1)} + m_n)/2]$$

where $m_{(n+1)}$ corresponds to the (n+1) mass value. Two successive mass differences are therefore plotted versus their corresponding mean values. The values obtained using equation (1) will be named "data" below.

A normalized cosine function is used for the fits of the "data":

$$\Delta M = \alpha(1 + cos((M - M_0)/M_1)) * exp(\beta.M)$$

where M/M_1 is defined within 2π. The oscillation periods, P = 2 πM_1 are studied. The amplitude of oscillations deserves theoretical studies which are outside the scope of the present work. M_0 depends on the mass choosen to start the fits, and is therefore arbitrary and suppressed. The fits are obtained with two fitted parameters when the oscillation amplitudes are constant (β=0) and three parameters when the oscillation amplitudes move progressively with a multiplicative value exp($\beta.M$). The formula used in some previous papers, to be compared with (2), was:

$$\Delta M = \alpha_0 + \alpha_1.cos((M - M_0)/M_1)$$

with again three adjustable parameters. A few "data" will be studied using this last relation, when the minimum of the fit is different from zero. When fitting the mass "data", ΔM corresponds to m(n+1) - m(n) and M to (m(n+1) + m(n))/2. For the width figures, ΔM corresponds to the width and M to the mass.

In order to be able to get an useful fit, the studies require at least the existence of several "data" in one arch. The extension of this study to heavier level masses is not possible as long as such mass ranges contain several levels without known quantum numbers (or widths).

Table 1. Parameters describing the fits of fig. 1. See text.

Fig.	α	β (MeV^{-1})	M$_C$ (MeV)	P (MeV)
1(a)	650 MeV	-0.0011	9600	766.5
1(b)	1 keV	-0.0009	9300	615.8
1(c)	433.5 MeV	-0.0012	3300	615.8
1(d)	4.7 keV	-0.002	3100	433.5

As already mentionned, it was previously said that this oscillatory property is often observed, when a given property is plotted versus another property correponding to the same studied familly. This will be illustrated below. In this case, the M and ΔM in relation (2) are replaced by "data" corresponding to the studied properties in figs. 1(a) and 1(c), and widths and masses in figs. 1(b) and 1(d).

3 The Oscillation Symmetry in Electromagnetic Particle Properties

In my previous papers, I evidenced the oscillatory behavior of different properties, such as masses and widths, of high energy particles (hadrons, mesons) and nuclei excited states. This is examplified by fig. 1, associated to Table 1 for bottomonium and charmonium particles.

In the present paper I question if electromagnetic properties of the nuclei excited states are also depicting an oscillatory behavior.

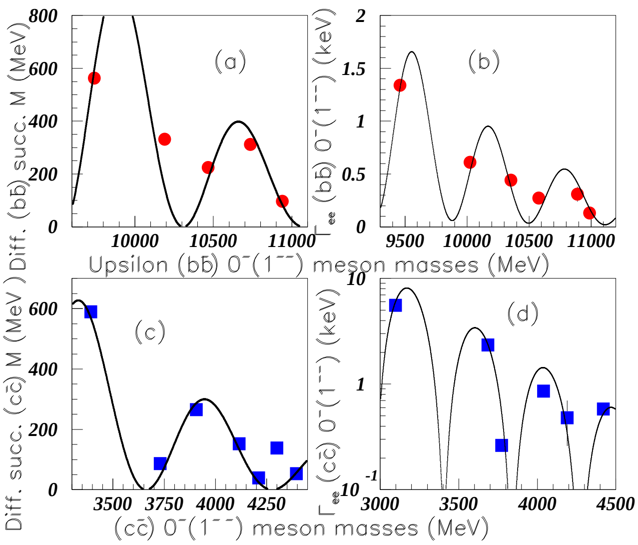

Figure 1. Color on line. "Data" of $0^-(1^{--})$ meson masses in (MeV). $(b\bar{b})$ in inserts (a) and (b), then $(c\bar{c})$ mesons in inserts (c) and (d). In inserts (a) and (c) the differences between successive masses are plotted versus their corresponding mean values. In inserts (b) and (d) the electromagnetic decay widths Γ_{ee}(keV) are plotted versus the corresponding masses.

Fig. 1 shows in insert (a) and (c), the Upsilon $b\bar{b}$ $0^-(1^{--})$ mass behavior and fit following relations (1) and (2) and the correponding "data" for the $c\bar{c}$ $0^-(1^{--})$ charmonium $(c\bar{c})$ masses (MeV). Figs. 1(b) and 1(d) show the electromagnetic decay widths Γ_{ee} (keV) [6] for $(b\bar{b})$ and $(c\bar{c})$ mesons versus the corresponding masses. The fit parameters are reported in Table 1. The corresponding "data" are not known for other meson masses. The periods are smaller for width ocillations than for mass oscillations. All data are well reproduced. The large mass values involve very small β values. In order to avoid loss of precision the $e^{\beta M}$ term is replaced by $e^{\beta(M-M_C)}$ term. M_C is arbitrary and therefore not considered to be an additionnal fitted parameter. The β parameters may be imprecise, except when they are small.

4 Electromagnetic Transitions Widths in Nuclei versus the Corresponding Mass Transitions

For data without known precisions, the imprecisions are arbitrarily introduced to 30/100 of the data value. All fits in this sect. are obtained using formula (2).

4.1 Study of the Electromagnetic Multipolarities with Use of Different Transitions between Different Initial and Final States

The magnetic data are drawn by red full circles, the electric data are drawn by blue full squares.

Since it is necessary to have enough data to extract oscillations, the electromagnetic widths must be studied for given multipolarities taking into account simultaneously the different transitions from excited nuclei level spins J_i to J_f. I start to check the validity of such assumption, by studying the M_1 periods observed for different transitions in ^{19}F and ^{18}F [7]. Indeed we have for these nuclei many data allowing to study separately different transitions.

Fig. 2 shows the M1 electromagnetic widths Γ_γ(meV) versus the transitions E_i-E_f (MeV) in ^{19}F [7]. Inserts (a) corresponds to $1/2^+$ to $3/2^+$ and $1/2^-$ to $3/2^-$ transitions and also to the inverse transitions. This will be noted $J_i\pm \leftrightarrow J_f\pm$. Inserts (b), (c), and (d) correspond respectively to $3/2\pm \leftrightarrow 5/2\pm$ transitions, $5/2\pm \leftrightarrow 7/2\pm$ transitions, and $7/2\pm \leftrightarrow 7/2\pm$ transitions.

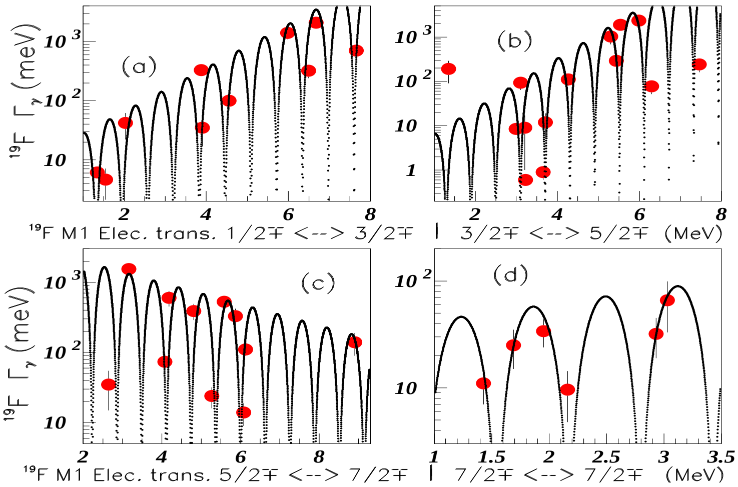

Figure 2. Color on line. Electromagnetic widths Γ_γ(meV) versus the E_i-E_f (MeV) transitions in ^{19}F. Inserts (a), (b), (c), and (d) select some different M_1 transitions. See text and Table 2.

Fig. 3 shows the electromagnetic widths Γ_γ(meV) versus the E_1 transitions E_i-E_f (MeV) in ^{18}F in inserts (a) and (b), and versus M_1 transitions in inserts (c) and (d) [7]. Inserts (a) corresponds to $2^\pm$ to $3^\mp$ transitions, insert (b) correspond to $2^\pm$ to $1^\mp$ transitions, insert (c) corresponds to $2^\pm$ to $3^\pm$ transitions, and insert (d) corresponds to $2^\pm$ to $1^\pm$ transitions.

The parameters of the fits are given in Table 2. In this Table the units for α are those of the ordinate, the units of β are the inverse of the abscissa units (the inverse of the period unit). We see that the periods P of the different M1 transitions in ^{19}F (Fig. 2) are nearly the same, allowing to incorporate several transitions in the same fig. for other nuclei having less known data. The periods of the studied M_1 transitions in ^{19}F (fig. 2) and ^{18}F (figs.3(c) and 3(d)) are also close to the mean value P = 0.624 MeV of the periods obtained in fig. 2 for ^{19}F nucleus.

The mean value of the periods of E_1 transitions in ^{18}F (figs.3(a) and 3(b)) is P = 0.6 MeV, close to individual values.

I will therefore in the following add the data corresponding to different transitions between excited levels and study separately the E_1 and M_1 transitions just as in a few cases the E_2 transitions.

A possible problem may arise. Although if generally the data can be fitted unambiguously, sometimes depending on the number of data and on their disposition, this is not the case. Such situation is illustrated in fig. 4 which shows the electromagnetic M_1 widths Γ_γ(eV) versus the transitions E_i-E_f (MeV) in ^{18}F [7]. Here the data corresponding to the M_1 transition 2^+ to 1^+ in ^{18}F are fitted with two different periods. The wide curve, blue on line, is obtained with P = 1.51 MeV, whereas the black on line curve is obtained using P = 0.625 MeV. This last value is preferred, since the corresponding fit is better. This value agrees with that of fig. 3(d). The quantitative informations concerning this fig. are reported in Table 2.

In order to clarify the following results of the paper, I illustrate the process by an example. The first reported electromagnetic M1 transition in ^{16}O is obtained through the excited state masses E_i = 8.87 MeV $J^P = 2^-$ and E_f = 6.13 MeV $J^P = 3^-$ [8]. The width of such transition is $\Gamma_\gamma = 3.0 \pm 0.4 \times 10^{-4}$ eV which corresponds to the transition transfer energy of 2.74 MeV. All M1 transitions in ^{16}O are collected and the widths versus the transition transfer energies are plotted. These "data" are fitted with the function (2), leading for each fig. to the determination of three fit parameters: α, β, and P.

4.2 Study of Oscillation Properties of Widths for Electromagnetic Transitions in Several Nuclei Data

Fig. 5 and 6 show several electromagnetic widths Γ_γ(eV) versus the transitions E_i-E_f (MeV) for fifteen nuclei from ^{8}Be to ^{20}Ne. Inserts (a), (b), and (c), correspond respectively to the E_1, M_1, and E_2 transitions

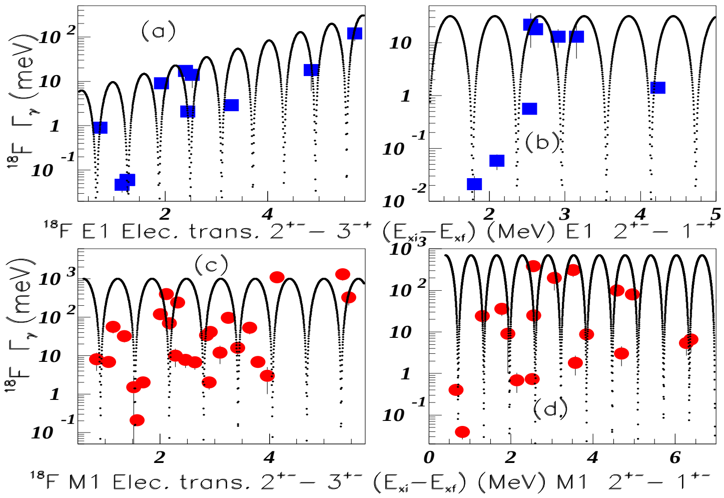

Figure 3. Color on line. Electromagnetic widths Γ_γ(meV) versus the E$_i$-E$_f$ (MeV) transitions in ^{18}F. Inserts (a) and (b) select some E$_1$ transitions, inserts (c) and (d) select some different M$_1$ transitions. See text and Table 2.

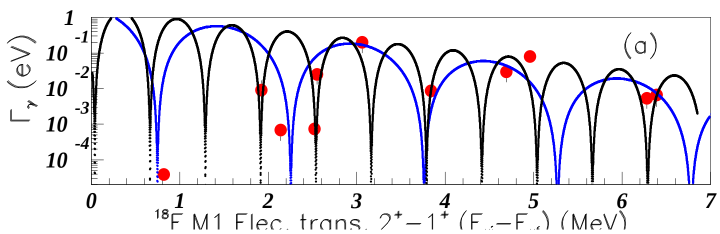

Figure 4. Color on line. Electromagnetic widths Γ_γ(eV) versus the transitions E$_i$-E$_f$ in (MeV). The M$_1$ transitions 2^+ to 1^+ in ^{18}F are fitted with two different periods (see text).

[7, 8, 9, 10, 11]. The extracted parameter values are given in Tables 3 and 4. The precision on the β values decreases for increasing β absolute values (increase of maxima slopes). M2 transitions are not shown due to the low number of data.

As an exemple, the list of transitions used to study the electromagnetic widths Γ_γ(eV) of ^{14}N [11] is detailed . Inserts (a) (line 7, column 1 in table 3) corresponds to the E$_1$ transitions: $0^- \to 1^+$, $2^- \to 1^+$, $1^- \to 0^+$, $0^+ \to 1^-$, $2^+ \to 3^-$, $3^+ \to 2^-$, a. so on.

Insert (b) (line 7, column 2 in table 3) corresponds to M$_1$ transitions: $2^+ \to 1^+$, $0^+ \to 1^+$, $1^+ \to 1^+$, $1+ \to 0^+$, $3^- \to 2^-$, $1^- \to 0^-$, $1^- \to 2^-$, $1^- \to 1^-$, $2^+ \to 3^+$, $2^+ \to 2^+$, a. so on.

Such detailed description of all used transitions will not be given from now. A large variation of the known number of transitions is observed for different nuclei. For ^{19}F [7], I used 25 first E$_1$ transitions, 30 first M$_1$ transitions, and 25 first E$_2$ transitions. Figs. 5 and 6 show that the amplitudes of the width ocillations increase in almost all cases (for the studied nuclei) with increase of the level energies. It does not mean that all widths increase. Their successive values all the time oscillate.

5 Study of Mass Oscillation Properties in Electromagnetic Transitions in Several Nuclei Data

The transition transfer energies are collected as before for the width studies, then classified in increasing order as has been done previously for hadrons and astrophysical "data", allowing to plot the differences between successive masses versus the corresponding mean values.

Table 2. Parameters of fits obtained for different M_1 transitions in ^{19}F and for different E1 and M1 multipolarities (Mult.) in ^{18}F.

Fig.	A	trans.	α	β	P
2(a)	^{19}F	1/2↔3/2	6 meV	0.85 MeV^{-1}	0.63 MeV
2(b)	^{19}F	3/2↔5/2	0.9 meV	1.3 MeV^{-1}	0.603 MeV
2(c)	^{19}F	5/2↔7/2	2000 meV	-0.35 MeV^{-1}	0.63 MeV
2(d)	^{19}F	7/2↔7/2	15 meV	0.35 MeV^{-1}	0.63 MeV
3(a)	^{18}F	2± ↔3∓	2.4 meV	0.71 MeV^{-1}	0.61 MeV
3(b)	^{18}F	2± ↔1∓	16 meV	0	0.59 MeV
3(c)	^{18}F	2± ↔3±	500 meV	0	0.628 MeV
3(d)	^{18}F	2± ↔1±	350 meV	0	0.626 MeV

Fig.	A	Mult.	α	β	P
4	^{18}F	M1	0.85 eV	-0.65 MeV^{-1}	0.63MeV
4	^{18}F	M1	0.85 eV	-0.75 MeV^{-1}	1.51MeV

As before, the "data" corresponding to E1 transitions are drawn by full blue squares, and the "data" corresponding to M1 transitions are drawn by full red circles.

The resulting "data" are shown in figs. 7 and 8. The oscillations are clearly observed. All fits are obtained using eq. (2). The quantitative informations corresponding to these two figs. are given in Tables 5 and 6.

There is any sens to study the variation of the α parameters, since they depend on the lowest mass of the known "data" and on the β parameter values. This is not the case for β parameters and periods P which values are discussed now. As already mentioned, the β parameters are specially imprecised for their relatively large (absolute) values. The variation of the β and P parameters is shown in figs. 9, and 10. The fits of figs. 9(a), 10(a), and 10(c) are obtained with the following relation containing four adjusted parameters: $\Delta(M) = (\alpha_0 + \alpha_1 cos((M - M_0)/M_1)) * exp(\beta.M)$ The fits of the "data" reported in other figs. are obtained using formula (3) with three parameters. All parameters are reported in Table 7.

Fig. 9 shows in insert (a) the variation of the previously extracted periods of the electromagnetic transition mass oscillations, plotted versus the atomic mass A. Fig. 9(a) shows that the amplitudes of the mass oscillation periods decrease for increasing atomic number nuclei. This reflects the decrease of mass separations between excited levels, for increasing A. However it does not mean that the "data" do not oscillate. Fig. 9(b) shows the periods of the widths parameters plotted versus the atomic mass A. Fig. 9(c) shows the ratio of the mass periods versus the width periods.

Fig. 10(a) shows the variation of the β parameter from mass oscillations corresponding to the E1 and M1 transitions versus the atomic number A. Fig. 10(b) shows the variation of the β parameter from width oscillations versus the atomic number A. Fig. 10(c) shows the variation of the ratio between β mass over β width versus the atomic number A. All these last three figs. which describe the periods of E1 and M1, exhibit oscillations and are fitted with eqs. (2) or (4) indicated with the fit parameters in Table 7. The mass periods of several nuclei are often close to 2 A. The ratios of mass over width periods are often close to 1. The same distribution describes the E1 and M1 "data".

6 Neutral Atomic Level Energies

The same study is applied to the neutral atomic level energies [12]. All references are obtained by substituting in reference [12] the atomic name under study to "atoms". I keep the data of the first spin J level of every configuration. Inside every configuration, the difference of these data, for increasing J, reduces in a large amount for increasing configurations.

Fig. 11 shows in inserts (a), (b), (c), (d), (e), and (f) the "data" and fits for hydrogen, ^{4}He, ^{7}Li, ^{9}Be, ^{12}C, and ^{14}N. These "data" are well fitted by equation (2), specially in inserts (a), (b), and (c). The

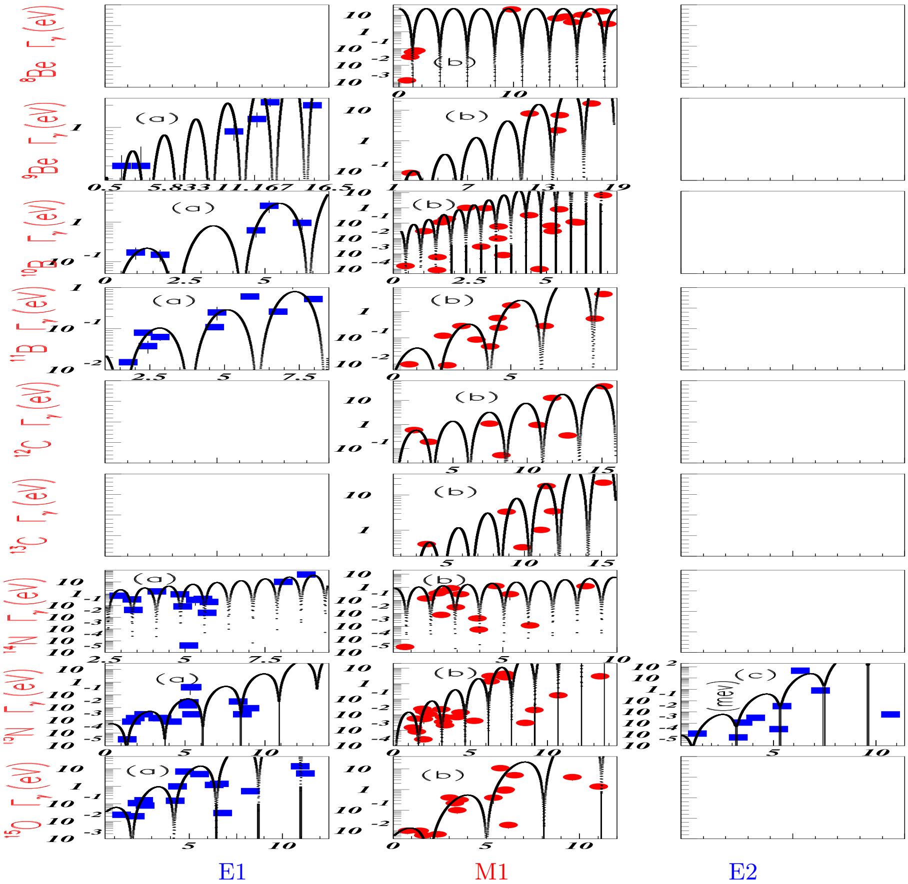

Figure 5. Color on line. Electromagnetic widths Γ_{γ}(meV) versus the E_1, M_1, and E_2, transitions E_i-E_f (MeV). The quantitative fitted parameters are given in Table 3.

corresponding parameters are reported in Table 8. The units of the "data" are cm^{-1} (1 meV=8.065 cm^{-1}). Some very low "data", as in fig.11(a), are not introduced otherwise the usefull range will be squeezed.

Fig. 12 shows in inserts (a), (b), (c), (d), (e), and (f) the "data" and fits for larger mass atoms, namely ^{16}O, ^{20}Ne, ^{22}Na, ^{27}Al, ^{28}Si and ^{31}P respectively.

We observe on figs. 11 and 12 that the oscillation amplitudes decrease with increasing atomic energy levels for all twelve studied atoms. We observe also that the oscillation periods shown in table 8 exhibit irregular behaviours. These periods, multiplied arbitrarily by 1.5 10^6, are plotted in fig.13 versus the atomic mass number, by full blue squares. This factor is introduced to be able to compare these results to the periods P of similar studies on nuclei.

Fig. 13 shows that the periods of the atomic level energies agree with those discussed before, and also with the periods of nuclear excited levels. The periods in fig. 13 are in MeV:

- full blue squares markers correspond to the atomic oscillating periods multiplied by 1.5 10^6,
- full green triangles show nuclear electric periods,
- full black stars show nuclear magnetic periods,
- full red markers show oscillating periods of nuclear excited J=2 levels,
- full red markers encircled by black empty squares correspond to oscillating periods of nuclear J=1 levels,
- full red markers encircled by black empty circles correspond to oscillating periods of nuclear J=5/2 levels,
- a full red marker encircled by black empty diamon-shaped marker corresponds to oscillating periods of nuclear J=7/2 levels.

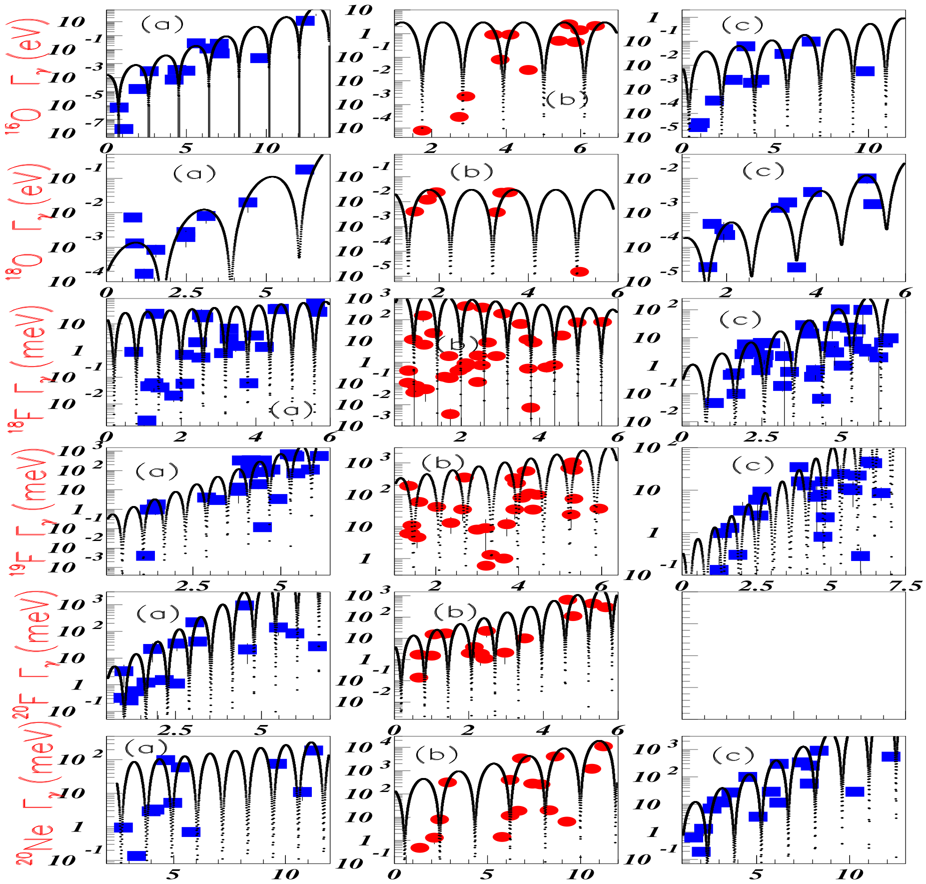

Figure 6. Color on line. Electromagnetic widths Γ_γ(meV) (or Γ_γ(eV)) versus the E_1, M_1, and E_2, transitions E_i-E_f (MeV). The quantitative fitted parameters are given in Table 4.

These nuclear oscillating period studies have been reported in [2] (and references inside) and [1]. All "data" in fig. 13 are well fitted by the same distribution PP=2.6 A, $\beta\beta$=-0.035 A^{-1}.

7 Conclusion

The main contribution of this work is the observation of the validity of the oscillation symmetry, namely that the experimental data oscillate.

The oscillation symmetry is applied to electromagnetic interactions E1, M1, (and E2 when the data are known) between different nuclei excited levels. A good agreement is generally observed between "data" and fits, obtained with two or three parameters, in the study of the widths and the masses of the transitions. These data are fitted with a simple oscillating function, although there is no known theory which justify the use of a simple cosine function. There is neither any known argument to observe oscillations in data, except for masses.

The extracted periods P and slope parameters β exhibit also oscillating shapes, the same for electric and magnetic transitions.

The oscillation symmetry is then applied to neutral atomic level energies. It is shown that again the studied "data" exhibit nice oscillations.

The same oscillation of the periods versus the atomic number A is observed for atomic and nuclear "data". The oscillation amplitudes of atomic "data" are smaller by a factor F≈ 1.5 10^6.

Table 3. Quantitative informations concerning the fits of fig. 5 which studies the widths oscillations in electromagnetic interactions in several nuclei. L, C means line and column numbers. Mult. means either E1, M1, or E2 multipolarities.

L, C	A	Mult.	α	β	P
1, 2	^{8}Be	M1	12 eV	0	2.39 MeV
2, 1	^{9}Be	E1	0.14 eV	0.22 MeV^{-1}	2.26 MeV
2, 2	^{9}Be	M1	0.014 eV	0.5 MeV^{-1}	2.51 MeV
3, 2	^{10}B	M1	0.015 eV	1.5 MeV^{-1}	0.49 MeV
4, 1	^{11}B	E1	0.015 eV	0.45 MeV^{-1}	2.26 MeV
4, 2	^{11}B	M1	0.0085 eV	0.95 MeV^{-1}	2.20 MeV
5, 2	^{12}C	M1	0.12 eV	0.35 MeV^{-1}	2.45 MeV
6, 2	^{13}C	M1	0.032 eV	0.51 MeV^{-1}	1.88 MeV
7, 1	^{14}N	E1	0.27 eV	0.47 MeV^{-1}	0.75 MeV
7, 2	^{14}N	M1	0.4 eV	0.2 MeV^{-1}	1.07 MeV
8, 1	^{15}N	E1	0.0002 eV	0.95 MeV^{-1}	2.38 MeV
8, 2	^{15}N	M1	0.0025 eV	1.1 MeV^{-1}	1.51 MeV
8, 3	^{15}N	E2	3.10^{-8} eV	1.95 MeV^{-1}	2.14 MeV
9, 1	^{15}O	E1	0.01 eV	1.2 MeV^{-1}	2.26 MeV
9, 2	^{15}O	M1	0.003 eV	1.2 MeV^{-1}	3.08 MeV

In conclusion, these results reinforce the previous observations showing that the oscillations are a general property often observed in nature.

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Table 4. Quantitative informations concerning the fits of the fig. 6 which studies the widths oscillations in electromagnetic interactions in several nuclei. L, C means line and column numbers. Mult. means either E1, M1, or E2 multipolarities.

L, C	A	Mult.	α	β	P
1, 1	$^{16}0$	E1	0.0009 eV	0.84 MeV^{-1}	1.88 MeV
1, 2	$^{16}0$	M1	1.5 eV	0	1.09 MeV
1, 3	$^{16}0$	E2	0.013 eV	0.3 MeV^{-1}	1.76 MeV
2, 1	$^{18}0$	E1	0.0003 eV	1.03 MeV^{-1}	2.14 MeV
2, 2	$^{18}0$	M1	0.015 eV	0	0.94 MeV
2, 3	$^{18}0$	E2	3.10^{-5} eV	1.04 MeV^{-1}	1.01 MeV
3, 1	^{18}F	E1	0.012 eV	0.18 MeV^{-1}	0.60 MeV
3, 2	^{18}F	M1	0.88 eV	-0.3 MeV^{-1}	0.60 MeV
3, 3	^{18}F	E2	0.0004 eV	1 MeV^{-1}	0.91 MeV
4, 1	^{19}F	E1	0.0002 eV	1.5 MeV^{-1}	0.60 MeV
4, 2	^{19}F	M1	0.1 eV	0.45 MeV^{-1}	0.63 MeV
4, 3	^{19}F	E2	0.0002 eV	1.1 MeV^{-1}	0.57 MeV
5, 1	^{20}F	E1	0.00072 eV	1.7 MeV^{-1}	0.63 MeV
5, 2	^{20}F	M1	0.004 eV	1.0 MeV^{-1}	0.63 MeV
6, 1	^{20}Ne	E1	0.025 eV	0.17 MeV^{-1}	1.13 MeV
6, 2	^{20}Ne	M1	0.12 eV	0.4 MeV^{-1}	1.88 MeV
6, 3	^{20}Ne	E2	0.00016 eV	0.8 MeV^{-1}	1.45 MeV

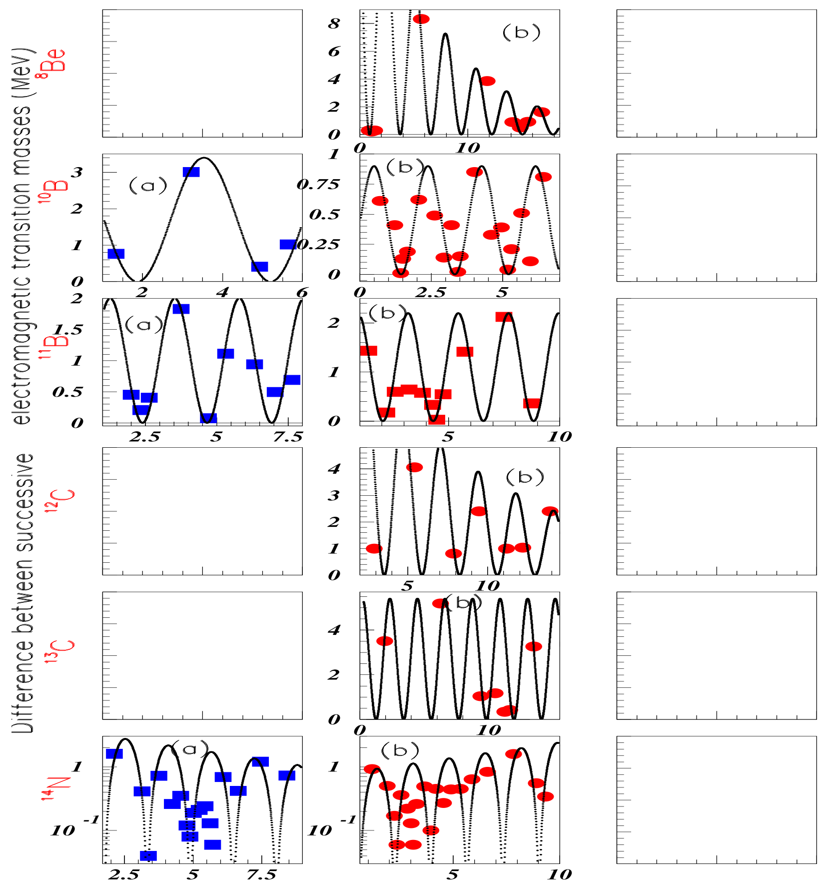

Figure 7. Color on line. Differences between successive masses versus the corresponding mean values. Inserts (a) and (b), show respectively the results for E1 and M1 transitions.

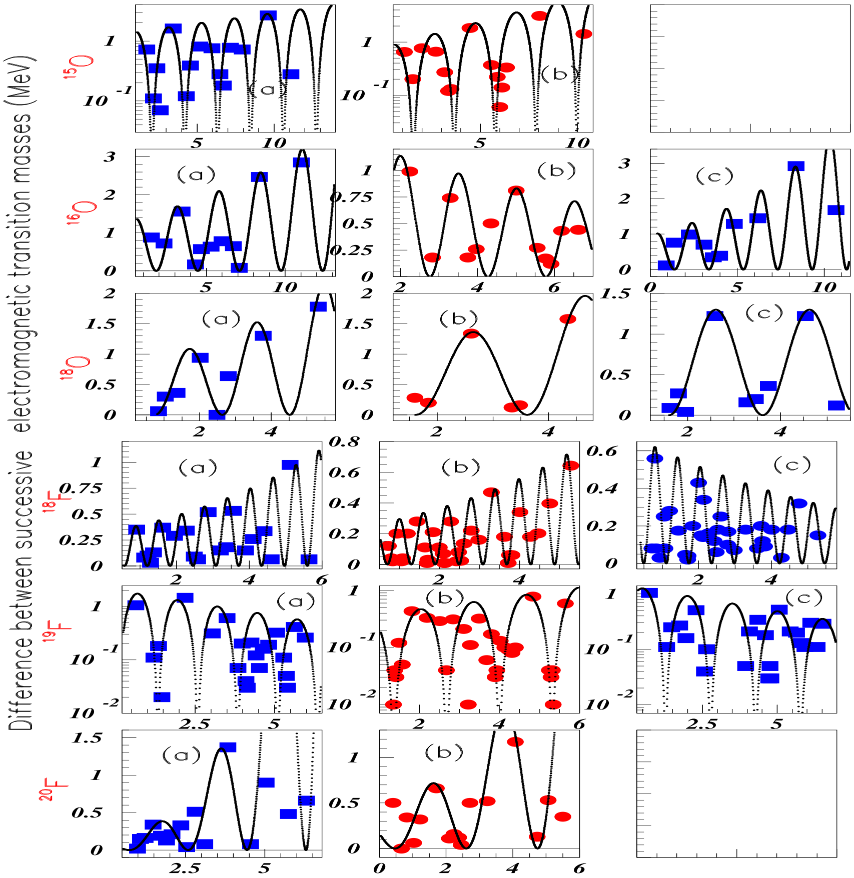

Figure 8. Color on line. Differences between successive masses versus the corresponding mean values. Inserts (a), (b), and (c) show respectively the results for E1, M1, and E2, transitions.

Table 5. Quantitative informations concerning the fits of fig. 7 which studies the mass oscillations in electromagnetic interactions in several nuclei. L, C means line and column numbers. Mult. means either E1, or M1 multipolarities.

L, C	A	Mult.	α	β	P
1, 2	^{8}Be	M1	12 MeV	-0.15 MeV^{-1}	2.83 MeV
2, 1	^{10}B	E1	1.7 MeV	0	3.33 MeV
2, 2	^{10}B	M1	0.45 eV	0	1.88
3, 1	^{11}B	E1	1.0 MeV	0	2.26 MeV
3, 2	^{11}B	M1	1.1 MeV	0	2.26 MeV
4, 2	^{12}C	M1	5.0 MeV	0.1 MeV^{-1}.	2.36 MeV
5, 2	^{13}C	M1	2.7 MeV	0	2.14 MeV
6, 1	^{14}N	E1	2.0 MeV	-0.15 MeV^{-1}	1.57 MeV
6, 2	^{14}N	M1	0.4 MeV	0.11 MeV^{-1}	1.7 MeV

Table 6. Quantitative informations concerning the fits of fig. 8 which studies the mass oscillations in electromagnetic interactions in several nuclei. L, C means line and column numbers. Mult. means either E1, M1, or E2 multipolarities.

L, C	A	Mult.	α	β	P
7, 1	^{15}O	E1	0.66 MeV	0.08 MeV^{-1}	2.14 MeV
7, 2	^{15}O	M1	0.4 MeV	0.22 MeV^{-1}	2.10 MeV
8, 1	^{16}O	E1	0.66 MeV	0.08 MeV^{-1}	2.64 MeV
8, 2	^{16}O	M1	0.7 MeV	-.104 MeV^{-1}	1.51 MeV
8, 3	^{16}O	E2	0.48 MeV	0.133 MeV^{-1}	1.98 MeV
9, 1	^{18}O	E1	0.40 MeV	0.18 MeV^{-1}	1.88 MeV
9, 2	^{18}O	M1	0.425 MeV	0.18 MeV^{-1}	2.01 MeV
9, 3	^{18}O	E2	0.65 MeV	0	2.01 MeV
10, 1	^{18}F	E1	0.16 MeV	0.211 MeV^{-1}	0.63 MeV
10, 2	^{18}F	M1	0.12 MeV	0.211 MeV^{-1}	0.625 MeV
10, 3	^{18}F	E2	0.35 MeV	-0.15 MeV^{-1}	0.62 MeV
11, 1	^{19}F	E1	1.0 MeV	-0.22 MeV^{-1}	1.26 MeV
11, 2	^{19}F	M1	0.15 MeV	0.22 MeV^{-1}	1.32 MeV
11, 3	^{19}F	E2	0.7 MeV	-0.21 MeV^{-1}	1.51 MeV
12, 1	^{20}F	E1	0.06 MeV	0.68 MeV^{-1}	1.85 MeV
12, 2	^{20}F	M1	0.2 MeV	0.37 MeV^{-1}	2.14 MeV

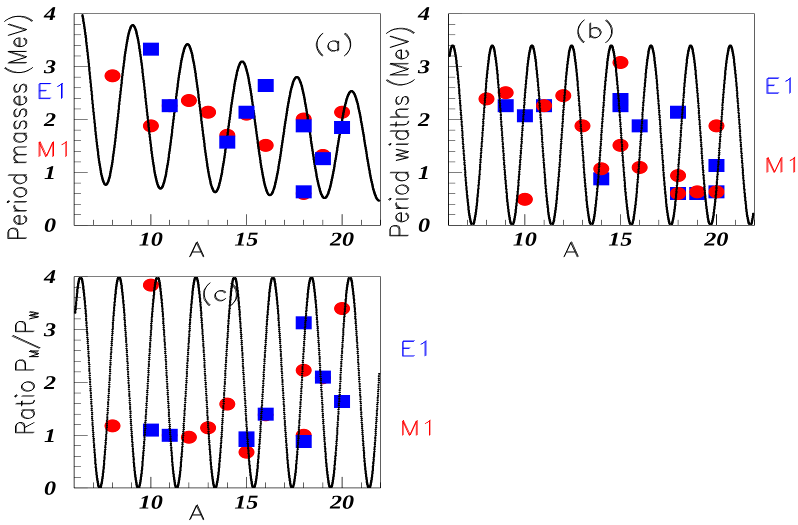

Figure 9. Color on line. Variation of the previously extracted periods of the electromagnetic transition mass oscillations, plotted versus the atomic mass A.

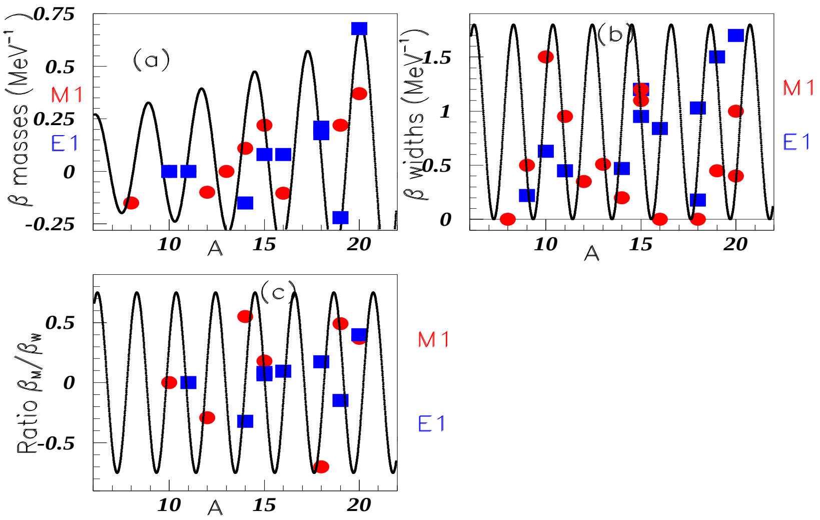

Figure 10. Color on line. Variation of the previously extracted periods of the β parameters describing the oscillations in the electromagnetic transitions, plotted versus the atomic mass A.

Table 7. Quantitative informations concerning the fits of previous figs. which study the mass oscillations in electromagnetic interactions of several nuclei.

Fig.	equa.	α	β	P	
9(b)	(2)	1.7 MeV	0	2.07 A	
9(c)	(2)	2 MeV	0	2.01 A	
10(b)	(2)	0.9 MeV^{-1}	0	2.07 A	
Fig.	equa.	α_0	α_1	β	P
9(a)	(4)	3.1 MeV	2.1 MeV	-0.035 A^{-1}	2.86 A
10(a)	(4)	0.03 MeV^{-1}	0.15 MeV^{-1}	0.067 A^{-1}	2.80 A
10(c)	(4)	0	0.75	0	2.07 A

Table 8. Quantitative informations concerning the fits of figs. 11 and 12 which study the oscillations of the atomic energy levels of several neutral atoms.

Fig.	A	α(cm^{-1})	β(cm)	P(cm^{-1})
11(a)	H	5.45 10^6	-7.4 10^{-5}	3393
11(b)	^{4}He	1.15 10^8	-5.75 10^{-5}	2513
11(c)	^{7}Li	1.9 10^4	-4.9 10^{-5}	5341
11(d)	^{9}Be	5.4 10^4	-5.3 10^{-5}	7571
11(e)	^{12}C	2.45 10^5	-6.5 10^{-5}	7414
11(f)	^{14}N	3.1 10^5	-4.5 10^{-5}	5341
12(a)	^{16}O	4.1 10^4	-2.4 10^{-5}	9739
12(b)	^{20}Ne	6.7 10^5	-3.1 10^{-5}	12535
12(c)	^{22}Na	6.9 10^4	-1.0 10^{-4}	4398
12(d)	^{27}Al	5.0 10^4	-9.7 10^{-5}	3016
12(e)	^{28}Si	3.0 10^4	-6.0 10^{-5}	12566
12(f)	^{31}P	1.45 10^5	-6.0 10^{-5}	12566

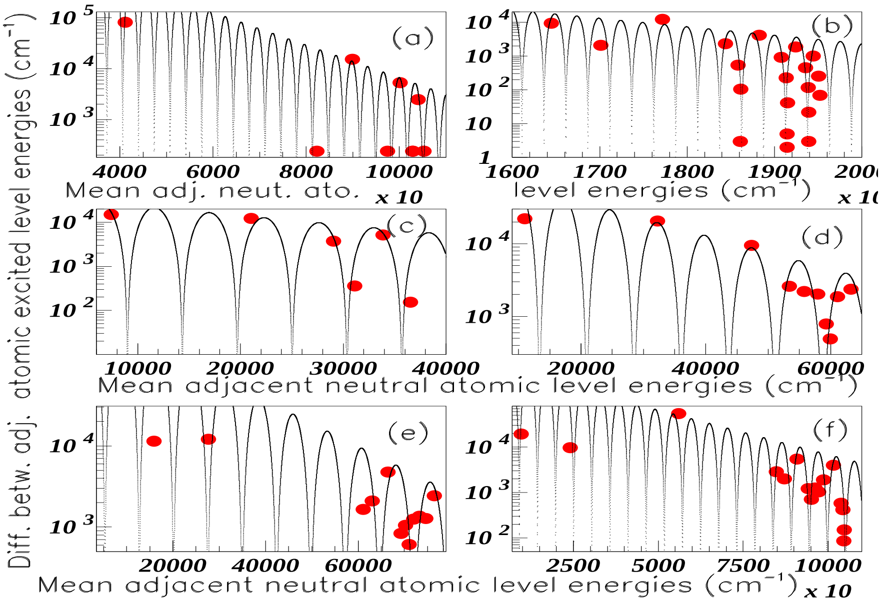

Figure 11. Color on line. Inserts (a), (b), (c), (d), (e), and (f) show the "data" and fits for hydrogen, ^{4}He, ^{7}Li, ^{9}Be, ^{12}C, and ^{14}N respectively.

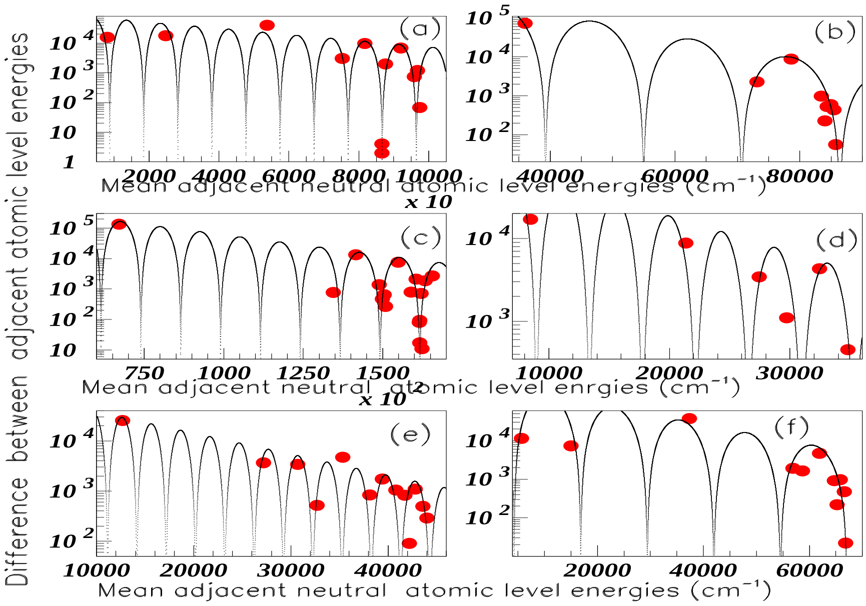

Figure 12. Color on line. Inserts (a), (b), (c), (d), (e), and (f) show the "data" and fits for ^{16}O, ^{20}Ne, ^{22}Na, ^{27}Al, ^{28}Si, and ^{31}P respectively.

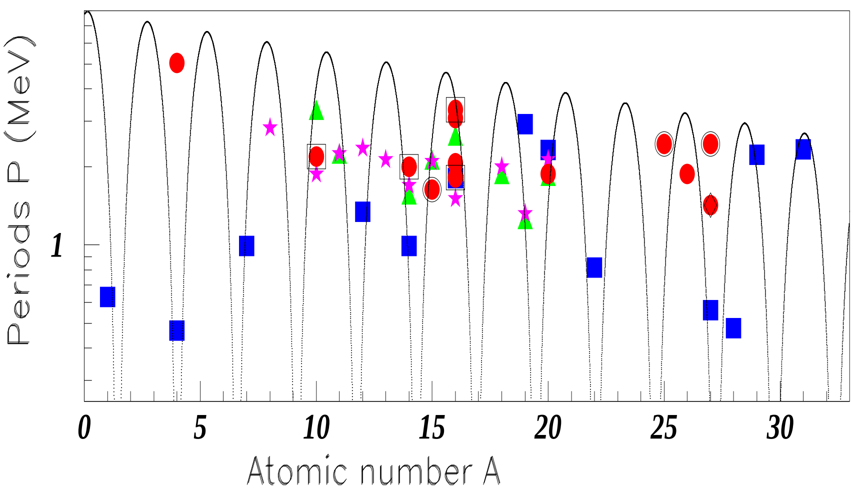

Figure 13. Color on line. Periods versus the atomic number A of atomic energy levels, of nuclear electric and magnetic desexcitation levels, and of nuclear excited state masses. See text.

