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Asymptotical Synchronization of Drive-Response Networks by Sample-Data-Based Event-Triggered Control with Quantization and Cyber-Attacks

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Abstract This paper addresses the asymptotic synchronization problem for a kind of drive-response complex networks (DRCNs) under cyber-attacks by using network control systems (NCSs). In order to reduce the pressure of communication and save the communication bandwidth on NCSs, some sampled-data-based event-triggered synchronization feedback controllers and logarithmic quantizers are designed by taking into account the effect of the NCSs' transmission delays. Using Lyapunov stability theories, several sufficient conditions are obtained to guarantee the existence of sampled-data-based event-triggered synchronization controllers for the DRCNs with distributed-delay. Then, the state feedback gains are obtained by solving certain linear matrix inequalities (LMIs). Finally, a numerical example is provided to illustrate the effectiveness of the sampled-data-based event-triggered control scheme.

Keywords: synchronization control; event-triggered mechanism; sampled-data-based; network control system; quantization; cyber-attack

1 Introduction

Over the past several years, complex dynamic networks (CDNs) has been fully studied owing to its frequent applications in many research areas, such as electric power grids, telecommunication networks and food webs. Up to now, considerable interesting studies have been reported on various problems for all kinds of CDNs [1,2,3,4]. In particular, synchronization is an universal phenomenon in CDNs and there has been an increasing amount of literature contributing to synchronization in arrays of CDNs [5,6,7,8]. Most of the research here have focused on synchronization among nodes in one network, namely internal synchronization. However, two (or more) systems could also achieve synchronization, which is external synchronization. [9] explain how different systems can achieve synchronization without considering the synchronization of the inner nodes. The synchronization of the drive-response system is an external synchronization, which means that the state of the response system follows the state of the drive system by controlling the response system. Therefore, the synchronization of drive-response systems has become the focus of researchers and achieved some recent results in [10,11].

In numerous control applications, controllers are implemented on continuous-time, such as pinning control scheme [12], complete synchronization scheme [13], and adaptive synchronization scheme [5]. However, with the rapid development of high-speed computers, modern control systems tend to be controlled by digital controllers, namely, only the samples of the control input signals at discrete time instants will be employed. Hence, the systems are always controlled by some discrete-time controllers in practical applications [4,14]. In order to reduce the difficulty of implementation and analysis, networked control systems (NCSs) have been proposed. The NCSs, as a capital type of complex dynamical systems, play an increasingly important role in the social infrastructures [15,16,17]. Nowadays, NCSs are widely used in various engineering fields due to their advantages such as flexibility, system efficiency and low maintenance cost. However, due to the limitation of communication bandwidth in the system, there are many unpredictable problems in NCSs, among these problems, network-induced delay [18], data packet dropouts [19] and multiple packets [20] are the most prominent. Because the communication resources in the network are very valuable, these defects will lead to the instability of the NCSs, which will also increase the difficulty of NCSs to perform control tasks. In order to keep the NCSs running smoothly and safely, an advanced methodology needs to be proposed to improve the bandwidth utilization of the communication channel, so as to offset the influence of network-induced delay, data packet and multiple packets caused by the network. Therefore, the event-triggered control (ETC) is concerned [21,22,23,24,25].

The basic idea of the ETC is that control tasks are executed when a well-designed event-triggered condition is met. That is the control tasks are executed only when needed. Hence, the ETC strategy can effectively reduce resource utilization and meanwhile ensure the desired levels of system performance, which motivates its wide application in numerous control issues. In [21], the authors investigated the event-triggered H_∞ controller design problem for NCSs by considering the effect of the network transmission delays. Then [22] developed the ETC method in [21] into a complex networks with uncertain inner coupling based on periodic sampling. In [25], an effective event-triggered state estimation scheme was proposed for a class of discrete-time multi-delayed neural networks. In addition, in order to avoid the Zeno behavior, a sampled-data-based event-triggered scheme was proposed and further developed in [26]. The sampled-data-based event-triggered scheme play an important role in ETC, and a number of results have been proposed in the literature [27,28]. Nevertheless, the synchronization control of event-triggered driven-response control networks (DRCNs) based on sampled data has not been studied and it is still a challenging problem.

In practical applications, signal transmission is usually limited by channel capacity and bandwidth. As another effective technology in term of improve the communication efficiency, the quantization has been widely applied in many systems, see, e.g. [29,30,31,32,33]. In [29], the control design problem for linear networked systems is investigated via event-triggered NCSs with quantization by using a Lyapunov functional. In [30], event-based control problem for nonlinear systems is concerned by using quantized control. As we know, there are usually two types of quantization, i.e. uniform quantizer and logarithmic quantizer, see [31,32,33]. In [32], the authors points out that the logarithmic quantizer performs superior to the uniform quantizer when it deal with the quantization error. But the signal quantization error may degrade sampled-data control systems performance severely, especially during in sampled-data systems, see [33]. Thus, in this case, it is necessary to study the sampled-data-based event-triggered control systems with quantization.

On the other hand, the security problems in NCSs have aroused much attention in the control community [18,19,20]. As the fact that a lot of signals need to be transmitted through networked communication channels, an offensive behavior named cyber-attacks is exposed. The purpose of the cyber-attacks is to exploit the vulnerabilities in the communication links to damage the data transmission systems, real-time sampling data, communication infrastructures and networked devices. At present, there are three major categories of cyber-attacks, that is denial of service (DoS) [34,35,36], replay attacks [37] and deception attacks [38]. Because of the huge impact of the cyber-attacks, large numbers of researchers have developed a strong interest in the study of cyber-attacks and have achieved many outstanding results [39,40,41]. The authors in [39] investigated online deception attack strategy with an event-triggered scheme to against remote state estimation. By considering the influence of quantization and deception attacks, the problem of distributed recursive filtering for a kind of discrete time-delay systems was investigated in [40]. Under the replay attacks, the authors studied a variation of the receding-horizon control in [41] and a set of sufficient conditions were provided to ensure asymptotical and exponential stability. So far, the synchronization problem for DRCNs with cyber-attacks has not been sufficiently discussed especially when the event-triggered synchronization controller design problem should be investigated more concretely for DRCNs with discrete-time delay and distributed-delay. Therefore, under measurement quantization and cyber-attacks, the synchronization control problem for sample-data-based event-triggered systems is still challenging.

Motivated by the aforementioned issues, in this paper, we will construct some event-triggered synchronization controllers for DRCNs with distributed-delay based on the periodic sampling via NCSs. Moreover, the effect of the measurement quantization and cyber-attacks are considered. The main contributions of this paper are summarized as follows: (i) A new synchronization error model for DRCNs is proposed in a integrated framework, which considering the influence of the cyber-attacks, measurement quantization and communication delays. (ii) In order to save the energy consumption, the sampled-data-based event-triggered scheme is used to determine whether the current sampled data is transmitted through the network, and the event-triggered synchronization controller for every node in the network is different with each other. (iii) The sufficient conditions are given to make sure that the existence of event-triggered synchronization controllers, and the event-triggered synchronization controller design problem can be solved perfectly by solving some LMIs. As far as we all know, there is no research investigating the asymptotic synchronization of sampled-data-based event-triggered control for DRCNs with cyber-attacks and quantization, so this is the main purpose of this paper.

Notations: In this paper, $\mathbb{R}^n$ respects the n dimensional Euclidean space with Euclidean norm, $\mathbb{R}^{n\times n}$ is the set of $n \times n$ real matrix. The notion $\|\cdot\|$ refers to Euclidean norm. Let $\mathbb{N} = \{0, 1, 2, \dots\}$ and $\mathbb{L} = \{1, 2, \dots, N\}$. For the matrix $\mathcal{D} \in \mathbb{R}^{n\times n}$, $\mathcal{D} > 0$ ($\mathcal{D} < 0$) is used to represent a positive (negative) definite matrix, denote by $\mathcal{D}^T$ and $\mathcal{D}^{-1}$ the transpose and inverse of the matrix $\mathcal{D}$, respectively. We use an asterisk * in a matrix to denote a term that is induced by symmetry. $E\{X\}$ denotes the expectation of stochastic variable X. $\mathrm{diag}\{A_1, A_2, \dots, A_N\}$ and $\mathrm{diag}_N\{A\}$ stand for a block-diagonal matrix with N blocks, which diagonal blocks are $A_i, i = 1, 2, \dots, N$ and A, respectively. $\otimes$ represents the Kronecker product of matrix. I denotes the identity matrix with proper dimension.

2 Problem Formulation and Preliminaries

Consider the following drive system with N nodes coupling and distributed-delay:

$$\begin{aligned}\dot{x}_i(t) = v(t, x_i(t)) + c_1\sum_{j=1}^{N}\alpha_{ij}\mathcal{M}_1 x_j(t) + c_2\sum_{j=1}^{N}\beta_{ij}\mathcal{M}_2 x_j(t-\varsigma)\\ + c_3\sum_{j=1}^{N}\gamma_{ij}\mathcal{M}_3\int_{t-\varsigma}^{t} x_j(s)ds, \ \text{for } i \in \mathbb{L},\end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the driven state vector of the ith node. $v : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuously differentiable vector function with $v(t, 0) \equiv 0$, for $t \geq 0$. $c_j > 0$ are the coupling strengths for $j = 1, 2, 3$. $\varsigma > 0$ is a constant delay. $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3 \in \mathbb{R}^{n\times n}$ are the inner coupling matrix, the delay inner coupling matrix and the distributed-delay inner coupling matrix, respectively. $\mathcal{A} = [\alpha_{ij}]_{N\times N}, \mathcal{B} = [\beta_{ij}]_{N\times N}$ and $\mathcal{C} = [\gamma_{ij}]_{N\times N}$ are the weight configuration matrices. $\alpha_{ij} > 0, \beta_{ij} > 0$, $\gamma_{ij} > 0$ if and only if there exists a link from node j to node i for any $i, j \in \mathbb{L}$.

The corresponding response system of the drive system (1) is defined as following

$$\begin{aligned}\dot{y}_i(t) = v(t, y_i(t)) + c_1\sum_{j=1}^{N}\alpha_{ij}\mathcal{M}_1 y_j(t) + c_2\sum_{j=1}^{N}\beta_{ij}\mathcal{M}_2 y_j(t-\varsigma)\\ + c_3\sum_{j=1}^{N}\gamma_{ij}\mathcal{M}_3\int_{t-\varsigma}^{t} y_j(s)ds + u_i(t), \ \text{for } i \in \mathbb{L},\end{aligned} \tag{2}$$

where $y_i(t) \in \mathbb{R}^n$ is the response state vector of the ith node, and other parameters are defined the same as the drive system (1), $u_i(t)$ is the control protocol for the ith node.

Remark 1 *We assume that each system has N nodes, and the nodes of the two systems correspond one-to-one, that is, each node in the drive system corresponds to the corresponding node in the response system, and the nodes in different systems have different dynamic characteristics.*

Let the synchronization error be $e_i(t) = y_i(t) - x_i(t)$ for $i \in \mathbb{L}$. Then the corresponding error system of networks (1) and (2) is given by

$$\begin{aligned}\dot{e}_i(t) = \tilde{v}(t, e_i(t)) + c_1\sum_{j=1}^{N}\alpha_{ij}\mathcal{M}_1 e_j(t) + c_2\sum_{j=1}^{N}\beta_{ij}\mathcal{M}_2 e_j(t-\varsigma)\\ + c_3\sum_{j=1}^{N}\gamma_{ij}\mathcal{M}_3\int_{t-\varsigma}^{t} e_j(s)ds + u_i(t), \ \text{for } i \in \mathbb{L},\end{aligned} \tag{3}$$

where $\tilde{v}(t, e_i(t)) = v(t, y_i(t)) - v(t, x_i(t))$.

In this paper, we denote the constant sampling period by h_i on the ith node, i.e. the sampling instants can be described as lh_i for $l \in \mathbb{N}$. As we know, in practical application, many "unnecessary" signals, which will waste the network communication resources and increase network transmission load, are sent by periodic sampling mechanism. Hence, an event generator is necessary to be introduced to determine which sampling signal should be sent out. For each node, we construct an event generator between the sampler and the NCSs to determine a release time sequence $\{t_k^i h_i\}_{k=0}^n$ with $t_0^i = 0$, which denotes the time when the newly sampled data are updated to the NCSs. Moreover, the next event-time instant $t_{k+1}^i h_i$ can be determined by

$$\begin{aligned} t_{k+1}^i = t_k^i + \min_{j\geq 1}\{j|(e_i((t_k^i+j)h_i) - e_i(t_k^i h_i))^T O_i \cdot (e_i((t_k^i+j)h_i) - e_i(t_k^i h_i)) \\ > \sigma_i e_i^T((t_k^i+j)h_i) O_i e_i((t_k^i+j)h_i)\}, \end{aligned} \tag{4}$$

for $k \in \mathbb{N}$, where $O_i \in \mathbb{R}^{n\times n}$ is a positive definite matrix and $\sigma_i \in [0,1]$ is a threshold parameter. Obviously, we have $0 = t_0^i < t_1^i < \cdots < t_k^i < \dots$ and $\{t_k^i h_i\}_{k=0}^n \subset \{kh_i\}_{k=0}^n$.

Remark 2 *By the event-trigger mechanism, only parts of sampled data will be transmitted to the controller. Thus the communication burden of the NCSs can be reduced, the transmission energy in NCSs can be saved and the lifespan of the battery of the NCSs also can be extended.*

Remark 3 *The frequency of transmitting sampled signals to the NCSs are determined by the threshold parameter σ_i. More concretely, the more smaller σ_i is chosen, the events can be triggered more easier.*

Remark 4 *Since the inter-event times are longer than the constant sampling period h_i for each node $i \in \mathbb{L}$, the Zeno-behavior can be avoid absolutely.*

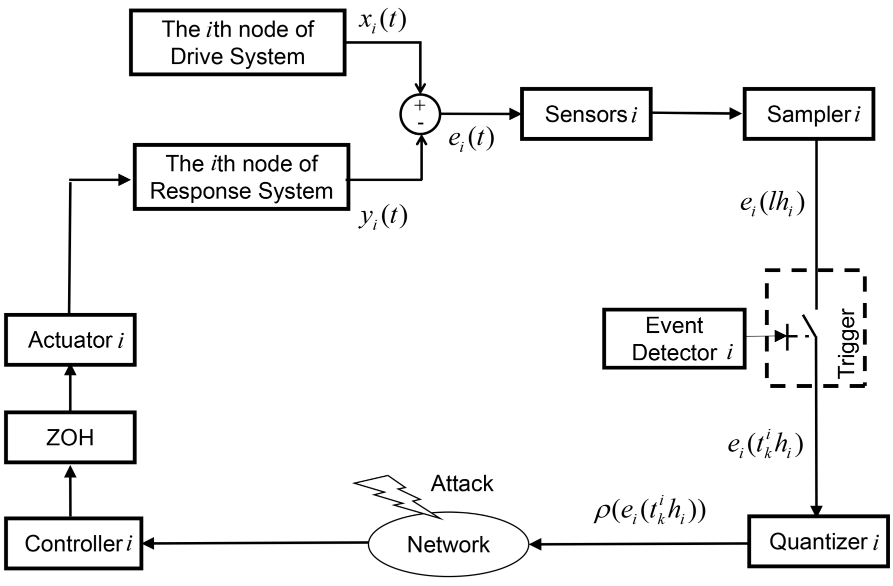

Figure 1. Flow chart of the quantization event-triggered mechanism.

The NCSs have high requirements for the bandwidth and frequency of data transmission. For the sake of further reducing the load of data transmission and improving the capability of networks, a quantizer is added between the event-triggered mechanism and the network (see Fig. 1). A static logarithmic quantizer $\rho(\cdot)$ is placed, which is defined as $\rho(\vartheta) = [\rho_1(\vartheta_1), \rho_2(\vartheta_2), ..., \rho_n(\vartheta_n)]^T, \forall \vartheta \in \mathbb{R}^n$, where for each

$\rho_j(\cdot)(1 \leq j \leq n)$ is the jth quantizer with the quantized density which is denoted by ρ_j^i with $0 < \rho_j < 1$, for $j = 1, 2, ..., n$, $i = 0, \pm1, \pm2, ...$ Then, the set of quantization levels are defined as following

$$\mathcal{N}_j = \{\pm n_i^{(j)} : n_i^{(j)} = \rho_j^i n_0^{(j)}, i = 0, \pm1, \pm2, ...\} \bigcup \{0\},$$

with $n_0^{(j)} > 0$. Moreover, the jth logarithmic quantizer of $\rho(\cdot)$ is defined as

$$\rho_j(d) = \begin{cases} n_i^{(j)}, & \frac{1}{1+\mu_j} n_i^{(j)} < d < \frac{1}{1-\mu_j} n_i^{(j)}, \\ 0, & d = 0, \\ -\rho_j(-d), & d < 0, \end{cases}$$

where $\mu_j = (1 - \varrho_j)/(1 + \varrho_j)$. Using the sector bound method [32], we can get the following quantization error:

$$\rho_j(d) - d = \triangle_j(d)d,$$

where$|\triangle_j(d)| \leq \mu_j, d \in \mathbb{R}$. Then, we can obtain

$$\rho_j(d) = (1 + \triangle_j(d))d.$$

Let $\triangle(D) = \text{diag}_n\{\triangle_i(D_i)\}$ for $D \in \mathbb{R}^n$. Consequently, the quantized state can be rewritten as

$$\rho(D) = (I + \triangle(D))D. \tag{5}$$

Remark 5 *The event-triggered scheme is an effective approach to minimize the use of the communication resources. Quantization in control systems, as another effective method to save the network bandwidth, has become a hot research topic. In this paper, not only did we think over how to reduce the use of the communication resources by adopting the event-triggering scheme, but also consider the effect of quantization.*

Remark 6 *There are two main types of quantization in the existing literatures, that is logarithmic quantization and uniform quantization [31]. The uniform quantizer is a useful mechanism because of its simplicity, but it suffers from dead-zone area, which increases proportionally with the quantization level. In comparison to that, the magnitude of the logarithmic quantization error is multiplicative, and decrease as the input signal becomes small. Due to this advantage, logarithmic quantization is a more effective means to save the network bandwidth in NCSs.*

In the following, the impact of transmission delays in NCSs will be considered. Suppose the transmission delay in the NCSs' communication is $\eta_k^i \in [0, \eta_i]$ for the ith node on the kth transmission. Hence, for each node i, the real control input can be defined as

$$u_i(t) = K_i\rho(e_i(t_k^i h_i)) = K_i(I + \triangle(e_i(t_k^i h_i)))e_i(t_k^i h_i),$$

for $t \in [t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i)$, where K_i denotes the feedback control gain matrix to be designed. For the convenience of writing, we replace $\triangle(e_i(t_k^i h_i))$ by $\triangle$ in the following, then the $u_i(t)$ can be rewritten as

$$u_i(t) = K_i\rho(e_i(t_k^i h_i)) = K_i(I + \triangle)e_i(t_k^i h_i), \tag{6}$$

for $t \in [t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i)$.

Taking similar discussion as [21], we divide the time interval into the following two cases for each $i \in \mathbb{L}$ and $k \in \mathbb{N}$:

Case 1. If $t_k^i h_i + h_i + \eta_i \geq t_{k+1}^i h_i + \eta_{k+1}^i$, define two functions as

$$\pi_i(t) = e(t_k^i h_i) - e(t_k^i h_i) = 0 \tag{7}$$

and

$$\eta_i(t) = t - t_k^i h_i, \text{ for } t \in [t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i). \tag{8}$$

Obviously, $\eta_k^i \leq \eta_i(t) \leq (t_{k+1}^i - t_k^i)h_i + \eta_{k+1}^i \leq h_i + \eta_i$

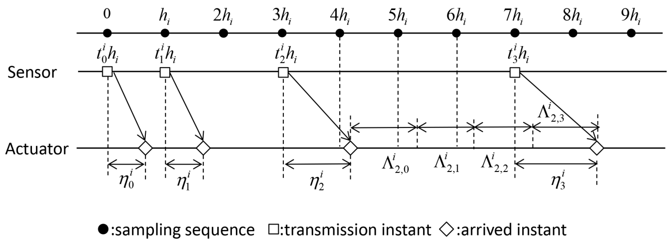

Figure 2. The time evolution of the sampling and transmission series for Case 2

Case 2. If $t_k^i h_i + h_i + \eta_i < t_{k+1}^i h_i + \eta_{k+1}^i$ (see Fig. 2). There exists a positive integer ζ_i such that

$$(t_k^i + \zeta_i)h_i + \eta_i \leq t_{k+1}^i h_i + \eta_{k+1}^i < (t_k^i + \zeta_i + 1)h_i + \eta_i.$$

Divide the time interval $[t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i)$ into the following form

$$[t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i) = \bigcup_{j=0}^{\zeta_i} \Lambda_{k,j}^i, \tag{9}$$

where

$$\begin{cases} \Lambda_{k,0}^i = [t_k^i h_i + \eta_k^i, (t_k^i + 1)h_i + \eta_i), \\ \Lambda_{k,m_i}^i = [(t_k^i + m_i)h_i + \eta_i, (t_k^i + m_i + 1)h_i + \eta_i), \\ \Lambda_{k,\zeta_i}^i = [(t_k^i + \zeta_i)h_i + \eta_i, t_{k+1}^i h_i + \eta_{k+1}^i), \end{cases}$$

for $m_i = 1, 2, ..., \zeta_i - 1$. We define

$$\eta_i(t) = t - (t_k^i + j)h_i \text{ for } t \in \Lambda_{k,j}^i \text{ and } j = 0, 1, ..., \zeta_i.$$

Obviously, it has

$$\begin{cases} \eta_k^i \leq \eta_i(t) \leq h_i + \eta_i, & t \in \Lambda_{k,0}^i, \\ \eta_k^i \leq \eta_i \leq \eta_i(t) \leq h_i + \eta_i, & t \in \Lambda_{k,j}^i, \end{cases}$$

for $j = 1, 2, ..., \zeta_i$. Therefore, we obtain

$$\eta_k^i \leq \eta_i(t) \leq \eta_M^i,$$

where $\eta_M^i = h_i + \eta_i$. Furthermore, we define the function

$$\pi_i(t) = e(t_k^i h_i) - e((t_k^i + j)h_i),$$

for $t \in \Lambda_{k,j}^i$ and $j = 0, 1, ..., \zeta_i$. Consequently, we have

$$\pi_i(t) = e(t_k^i h_i) - e(t - \eta_i(t)), t \in [t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i).$$

By the definition of $\eta_i(t)$ and $\pi_i(t)$ for $i \in \mathbb{L}$, the triggering algorithm in (4) and control input (6) can be rewritten as

$$\pi_i^T(t) O_i \pi_i(t) \leq \sigma_i e_i^T(t - \eta_i(t)) O_i e_i(t - \eta_i(t)), \tag{10}$$

$$u_i(t) = K_i(I + \triangle)\pi_i(t) + K_i(I + \triangle)e_i(t - \eta_i(t)), \tag{11}$$

for $t \in [t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i)$.

Remark 7 *The transmission speed from the NCS to the controller will be affected by the communication bandwidth, the environment of the device and so on. Therefore, it is naturally to consider the transmission delays.*

Regarding the openness of the NCS, some reasons, such as the controller signal is easy to theft, leakage or malicious damage, may lead the controller to fail to operate normally or issue wrong control instructions. Hence, we assume that the information transmitted through the communication network is fragile to be attacked. In this paper, the stochastic deception attack on the controller is considered. We define the attack signal as a non-linear function $\phi(u_i(t))$ with $\phi(0) = 0$, which is associated with the control input $u_i(t)$.

Let $\xi_i \in \{0, 1\}$ is employed to describe the randomly occurring possibility of cyber-attacks, which is a stochastic variable obeying the Bernoulli distribution with

$$P\{\xi_i = 1\} = \delta_i \text{ and } P\{\xi_i = 0\} = 1 - \delta_i.$$

Consequently, we can get

$$\mathbb{E}\{\xi_i\} = \delta_i \text{ and } \mathbb{E}\{(\xi_i - \delta_i)^2\} = \delta_i(1 - \delta_i) := \varpi_i^2,$$

where δ_i and ϖ_i^2 are the expectation and the mathematical variance of ξ_i, respectively.

Let $o_i(t) = \xi_i$, for $t \in [t_k^i h_i + \eta_k^i, t_{k+1}^i h_i + \eta_{k+1}^i)$. Considering the effect of cyber-attack in NCS, the control law in (11) can be expressed as follows

$$\begin{aligned}\bar{u}_i(t) &= u_i(t) + o_i(t)\phi(u_i(t)) \\ &= K_i(I + \triangle)(\pi_i(t) + e_i(t - \eta_i(t))) + o_i(t)\phi(u_i(t)).\end{aligned} \tag{12}$$

Remark 8 *According to the formula (12), when $o_i(t) = 1$, it means that the aggressive signals are delivered and the controller suffers from malicious attack signals, then the real control law in (12) can be rewritten as $\bar{u}_i(t) = u_i(t) + \phi(u_i(t))$. When $o_i(t) = 0$, means the network environment is secure, regardless of the impact of cyber-attacks in information transmission, the control law can be represented as $\bar{u}(t) = u_i(t)$.*

Let

$$\begin{aligned}
e(t) &= [e_1^T(t), e_2^T(t), ..., e_N^T(t)]^T, \\
\bar{e}(t) &= [\int_{t-\varsigma}^t e_1^T(s)ds, \int_{t-\varsigma}^t e_2^T(s)ds, ..., \int_{t-\varsigma}^t e_N^T(s)ds]^T, \\
e_{\eta_M}(t) &= [e^T(t - \eta_M^1), e^T(t - \eta_M^2), ..., e^T(t - \eta_M^N)]^T, \\
e(t - \eta(t)) &= [e_1^T(t - \eta_1(t)), e_2^T(t - \eta_2(t)), ..., e_N^T(t - \eta_N(t))]^T, \\
\pi(t) &= [\pi_1^T(t), \pi_2^T(t), ..., \pi_N^T(t)]^T, \\
\mathcal{V}(t, e(t)) &= [\tilde{v}^T(t, e_1(t)), \tilde{v}^T(t, e_2(t)), ..., \tilde{v}^T(t, e_N(t))]^T, \\
K &= \mathrm{diag}\{K_1, K_2, ..., K_N\}, \\
O &= \mathrm{diag}\{O_1, O_2, ..., O_N\}, \\
\Theta &= \mathrm{diag}_N\{\sigma_i\} \otimes I,\ \eta_M = \max_{i \in \mathbb{L}}\{\eta_M^i\}, \\
\bar{\triangle} &= \mathrm{diag}_N\{I + \triangle\},\ o(t) = \mathrm{diag}_N\{o_i(t)\} \otimes I, \\
\Phi(u(t)) &= [\phi^T(u_1(t), \phi^T(u_2(t))), ..., \phi^T(u_N(t))]^T.
\end{aligned}$$

Then, the error dynamics (3) can be expressed by the following compact form

$$\begin{aligned}\dot{e}(t) = &\mathcal{V}(t, e(t)) + c_1(\mathcal{A} \otimes \mathcal{M}_1)e(t) + c_2(\mathcal{B} \otimes \mathcal{M}_2)e(t - \varsigma) \\ &+ c_3(\mathcal{C} \otimes \mathcal{M}_3)\bar{e}(t) + K\bar{\triangle}(\pi(t) + e(t - \eta(t))) + o(t)\Phi(u(t)),\end{aligned} \tag{13}$$

for $t \geq 0$, and the triggering algorithm (10) can be rewritten as

$$\pi^T(t)O\pi(t) \leq e^T(t - \eta(t))\Theta O e(t - \eta(t)). \tag{14}$$

Definition 1 *The drive system (1) and the response system (2) are said achieve asymptotical synchronization if the synchronization error dynamical system (13) is asymptotically stable.*

The aim of this paper is to design suitable controller gain matrices K_i for $i \in \mathbb{L}$ such that the drive network (1) and the response network (2) achieve asymptotically synchronization, i.e., the asymptotical stability of the error dynamical system (13). In order to derive this results, we give the following assumptions and lemmas.

Assumption 1 *[42] The attack signal function $\Phi(\cdot)$ and the vector valued function $\mathcal{V}(\cdot)$ are assumed to be nonlinear functions, which satisfy the following Lipschitz constraints*

$$\|\Phi(x) - \Phi(y)\| \leq \|\lambda_1(x-y)\|, \tag{15}$$
$$\|\mathcal{V}(x) - \mathcal{V}(y)\| \leq \|\lambda_2(x-y)\|, \tag{16}$$

where λ_1 and λ_2 are constant matrices representing for the upper bounds of $\Phi(\cdot)$ and $\mathcal{V}(\cdot)$.

Lemma 1 *[22] Let E,S and H be real matrices with appropriate dimensions,and H satisfies $H^T H \leq I$. Then,for any scalar $\epsilon > 0$, the following inequality holds:*

$$E^T H S^T + S H^T E \leq \epsilon S S^T + \epsilon^{-1} E^T E. \tag{17}$$

Lemma 2 *[43] For given positive constant η_M, if function $\eta(t)$ satisfies $\eta(t) \in (0, \eta_M]$, then there exists $R > 0$ such that*

$$-\eta_M \int_{t-\eta_M}^{t} \dot{e}^T(s) R \dot{e}(s) \leq \begin{bmatrix} e(t) \\ e(t-\eta(t)) \\ e(t-\eta_M) \end{bmatrix}^T \begin{bmatrix} -R & R & 0 \\ * & -2R & R \\ * & * & -R \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\eta(t)) \\ e(t-\eta_M) \end{bmatrix}.$$

3 Event-triggered Controller Design

In this section, some appropriate controller gain matrices K_i will be designed for the response system (2) so that the drive network (1) and the response network (2) achieve asymptotic stability under the event-triggered strategy in (4).

Theorem 1 *Suppose Assumption 1 is satisfied. For the given controller gain matrices $K_i (i \in \mathbb{L})$, the drive network (1) and the response network (2) achieve asymptotically stable under the event-triggered strategy in (4), if there exist some positive matrices P, Q_i, R, T, O_i , for $i \in \mathbb{L}$ such that*

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} \\ * & -R & 0 & 0 \\ * & * & -R & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{18}$$

where

$$\Sigma_{11} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & \Xi_{15} & \Xi_{16} & P & P\bar{\delta} \\ * & -T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & R & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -O & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned} \Sigma_{12} &= \eta_M (R\mathcal{F})^T, \\ \mathcal{F} &= \begin{bmatrix} c_1(\mathcal{A}\otimes\mathcal{M}_1) \ c_2(\mathcal{B}\otimes\mathcal{M}_2) \ 0 \ 0 \ K\bar{\triangle} \ K\bar{\triangle} \ I \ \bar{\delta} \end{bmatrix}, \\ \Sigma_{13} &= \begin{bmatrix} 0\ 0\ 0\ 0\ 0\ 0\ 0\ \eta_M R\mathcal{W} \end{bmatrix}^T, \\ \Sigma_{14} &= \begin{bmatrix} 0\ 0\ 0\ 0\ \bar{\lambda}_1 K\bar{\triangle}\ \bar{\lambda}_1 K\bar{\triangle}\ 0\ 0 \end{bmatrix}^T, \\ \Xi_{11} &= c_1 P(\mathcal{A}\otimes\mathcal{M}_1) + c_1(\mathcal{A}\otimes\mathcal{M}_1)^T P + Q - R + T + \bar{\lambda}_2^T\bar{\lambda}_2, \\ \Xi_{12} &= c_2 P(\mathcal{B}\otimes\mathcal{M}_2), \quad \Xi_{13} = c_3 P(\mathcal{C}\otimes\mathcal{M}_3), \\ \Xi_{15} &= PK\bar{\triangle} + R, \quad \Xi_{16} = PK\bar{\triangle}, \\ \Xi_{33} &= \eta_M^2 c_3(\mathcal{C}\otimes\mathcal{M}_3)^T R c_3(\mathcal{C}\otimes\mathcal{M}_3), \\ \Xi_{44} &= -R - Q, \Xi_{55} = \Theta O - 2R, \\ \bar{\lambda}_1 &= diag_N\{\lambda_1\}, \bar{\lambda}_2 = diag_N\{\lambda_2\}, \\ \mathcal{W} &= diag_N\{\varpi_i\}\otimes I, \bar{\delta} = diag_N\{\delta_i\}\otimes I. \end{aligned}$$

proof 1 *Let*

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{19}$$

where

$$\begin{aligned} V_1(t) &= e^T(t)Pe(t), \\ V_2(t) &= \int_{t-\eta_M}^{t} e^T(s)Qe(s)ds, \\ V_3(t) &= \eta_M \int_{t-\eta_M}^{t}\int_{s}^{t} \dot{e}^T(w)R\dot{e}(w)dwds, \\ V_4(t) &= \int_{t-\varsigma}^{t} e^T(s)Te(s)ds. \end{aligned}$$

Calculating the time derivative of $V(t)$ along the error dynamics (13) and taking expectation on it, we have

$$\begin{aligned} \mathbb{E}\{\dot{V}_1(t)\} = 2e^T(t)P[&\mathcal{V}(t,e(t)) + c_1(\mathcal{A}\otimes\mathcal{M}_1)e(t) \\ &+ c_2(\mathcal{B}\otimes\mathcal{M}_2)e(t-\varsigma) + c_3(\mathcal{C}\otimes\mathcal{M}_3)\bar{e}(t) \\ &+ K\bar{\triangle}(\pi(t) + e(t-\eta(t))) + \bar{\delta}\Phi(u(t))], \end{aligned} \tag{20}$$

$$\mathbb{E}\{\dot{V}_2(t)\} = e^T(t)Qe(t) - e^T(t-\eta_M)Qe(t-\eta_M), \tag{21}$$

$$\mathbb{E}\{\dot{V}_3(t)\} = \mathbb{E}\{\eta_M^2 \dot{e}^T(t)R\dot{e}(t) - \eta_M\int_{t-\eta_M}^{t}\dot{e}^T(s)R\dot{e}(s)ds\}, \tag{22}$$

$$\mathbb{E}\{\dot{V}_4(t)\} = e^T(t)Te(t) - e^T(t-\varsigma)Te(t-\varsigma). \tag{23}$$

Notice that $\dot{e}(t) = \mathcal{X} + (o(t) - \bar{\delta})\varPhi(u(t))$, *where*

$$\begin{aligned}\mathcal{X} = \quad & \mathcal{V}(t, e(t)) + c_1(\mathcal{A} \otimes \mathcal{M}_1)e(t) + c_2(\mathcal{B} \otimes \mathcal{M}_2)e(t - \varsigma) \\ & + c_3(\mathcal{C} \otimes \mathcal{M}_3)\overline{e}(t) + K\bar{\triangle}(\pi(t) + e(t - \eta(t))) + \bar{\delta}\varPhi(u(t)).\end{aligned}$$

Then, we can obtain

$$\mathbb{E}\{\eta_M^2 \dot{e}^T(t)R\dot{e}(t)\} = \eta_M^2(\mathcal{X}^T R\mathcal{X} + \varPhi^T(u(t))\mathcal{W}^T R\mathcal{W}\varPhi(u(t))). \tag{24}$$

According to Assumption 1, we can derive

$$\varPhi^T(u(t))\varPhi(u(t)) \leq u^T(t)\bar{\lambda}_1^T\bar{\lambda}_1 u(t), \tag{25}$$

$$\mathcal{V}^T(t, e(t))\mathcal{V}(t, e(t)) \leq e^T(t)\bar{\lambda}_2^T\bar{\lambda}_2 e(t). \tag{26}$$

It follows from (20)-(26) and Lemma 2 that

$$\begin{aligned}
& \mathbb{E}\{\dot{V}(t)\} \leq 2e^T(t)P[\mathcal{V}(t, e(t)) + c_1(\mathcal{A} \otimes \mathcal{M}_1)e(t) \\
& + c_2(\mathcal{B} \otimes \mathcal{M}_2)e(t - \varsigma) + c_3(\mathcal{C} \otimes \mathcal{M}_3)\overline{e}(t) \\
& + K\bar{\triangle}(\pi(t) + e_\eta(t)) + \bar{\delta}\varPhi(u(t))] \\
& + e^T(t)Qe(t) - e^T(t - \eta_M)Qe(t - \eta_M) \\
& + \eta_M^2(\mathcal{X}^T R\mathcal{X} + \varPhi^T(u(t))\mathcal{W}^T R\mathcal{W}\varPhi(u(t))) \\
& + \begin{bmatrix} e(t) \\ e(t - \eta(t)) \\ e(t - \eta_M) \end{bmatrix}^T \begin{bmatrix} -R & R & 0 \\ * & -2R & R \\ * & * & -R \end{bmatrix} \begin{bmatrix} e(t) \\ e(t - \eta(t)) \\ e(t - \eta_M) \end{bmatrix} \\
& + e^T(t)Te(t) - e^T(t - \varsigma)Te(t - \varsigma) \\
& + e^T(t - \eta(t))\varTheta Oe(t - \eta(t)) - \pi^T(t)O\pi(t) \\
& + u^T(t)\bar{\lambda}_1^T\bar{\lambda}_1 u(t) - \varPhi^T(u(t))\varPhi(u(t)) \\
& + e^T(t)\bar{\lambda}_2^T\bar{\lambda}_2 e(t) - \mathcal{V}^T(t, e(t))\mathcal{V}(t, e(t)) \\
= \ & \zeta^T(t)[\varSigma_{11} + \eta_M^2\mathcal{F}^T R\mathcal{F}]\zeta(t) + \eta_M^2\varPhi^T(u(t))\mathcal{W}^T R\mathcal{W}\varPhi(u(t)) \\
& + u^T(t)\bar{\lambda}_1^T\bar{\lambda}_1 u(t) \\
= \ & \zeta^T(t)[\varSigma_{11} + \varSigma_{12}R^{-1}\varSigma_{12}^T + \varSigma_{13}R^{-1}\varSigma_{13}^T + \varSigma_{14}\varSigma_{14}^T]\zeta(t),
\end{aligned}$$

where

$$\begin{aligned}\mathcal{X} &= \mathcal{F}\zeta(t) + c_3(\mathcal{C} \otimes \mathcal{M}_3)\overline{e}(t), \\ \zeta(t) &= [e^T(t), e^T(t - \varsigma), \overline{e}^T(t), e_{\eta_M}^T, e_\eta^T(t), \pi^T(t), \mathcal{V}^T(t, e(t)), \varPhi^T(u(t))]^T.\end{aligned}$$

Consequently, by using Schur complement lemma and (18), *we have* $E\{\dot{\mathcal{V}}(t)\} < 0$. *Therefore, the error system* (13) *is asymptotically stable. This completes the proof.*

It is worth mentioning that Theorem 1 does not design a reasonable controller. In the following, we will deal with the synchronization controller design problem based on the stability criterion established in Theorem 1.

Theorem 2 *Suppose the Assumption 1 is satisfied. For given scalars* $\delta_i > 0, \eta_M > 0, \epsilon_j > 0$ $(j = 1, 2)$, *the drive network (1) and the response network (2) achieve asymptotically synchronization, if there exist matrices* $F = diag\{F_1, F_2, \ldots, F_N\} > 0$, $X = diag\{X_1, X_2, \ldots, X_N\} > 0$, $\bar{T} > 0$, $\bar{Q} > 0$, $\bar{O} > 0, \bar{R} > 0$ *such that*

$$\varPsi = \begin{bmatrix} \bar{\varSigma}_1 & \varPhi_{11} & \varPhi_{12} & \varPhi_{13} \\ * & (\epsilon_1 + \epsilon_2)(-2F + I) & 0 & 0 \\ * & * & -\epsilon_1\mu^2 I & 0 \\ * & * & * & -\epsilon_2\mu^2 I \end{bmatrix} < 0, \tag{27}$$

where

$$\bar{\varSigma}_1 = \begin{bmatrix} \bar{\varSigma}_{11} & \bar{\varSigma}_{12} & \bar{\varSigma}_{13} & \bar{\varSigma}_{14} & \bar{\varSigma}_{15} \\ * & -2F+\bar{R} & 0 & 0 & 0 \\ * & * & -2F+\bar{R} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix}, \tag{28}$$

$$\bar{\varSigma}_{11} = \begin{bmatrix} \bar{\varXi}_{11} & \bar{\varXi}_{12} & \bar{\varXi}_{13} & 0 & \bar{\varXi}_{15} & \bar{\varXi}_{16} & I & \bar{\delta} \\ * & -\bar{T} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \bar{\varXi}_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\varXi}_{44} & \bar{R} & 0 & 0 & 0 \\ * & * & * & * & \bar{\varXi}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{O} & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned}
\bar{\varXi}_{11} &= c_1(\mathcal{A}\otimes\mathcal{M}_1)F + c_1F(\mathcal{A}\otimes\mathcal{M}_1)^T + \bar{Q}\bar{R} + \bar{T}, \\
\bar{\varXi}_{12} &= c_2(\mathcal{B}\otimes\mathcal{M}_2)F, \quad \bar{\varXi}_{13} = c_3(\mathcal{C}\otimes\mathcal{M}_3)F, \\
\bar{\varXi}_{15} &= X+\bar{R}, \bar{\varXi}_{16} = X, \\
\bar{\varXi}_{33} &= \eta_M^2 c_3(\mathcal{C}\otimes\mathcal{M}_3)^T\bar{R}c_3(\mathcal{C}\otimes\mathcal{M}_3), \\
\bar{\varXi}_{44} &= -\bar{Q}-\bar{R}, \bar{\varXi}_{55} = \varTheta\bar{O}-2\bar{R}, \bar{\varSigma}_{12} = \eta_M\bar{\mathcal{F}}^T, \\
\bar{\varSigma}_{13} &= \begin{bmatrix}0\ 0\ 0\ 0\ 0\ 0\ 0\ \eta_M\mathcal{W}\end{bmatrix}^T, \\
\bar{\varSigma}_{14} &= \begin{bmatrix}0\ 0\ 0\ 0\ \bar{\lambda}_1X\ \bar{\lambda}_1X\ 0\ 0\end{bmatrix}^T, \\
\bar{\varSigma}_{15} &= \begin{bmatrix}\bar{\lambda}_2F\ 0\ 0\ 0\ 0\ 0\ 0\ 0\end{bmatrix}^T, \\
\varPhi_{11} &= \begin{bmatrix}(\epsilon_1+\epsilon_2)X^T\ 0_{\bar{N}\times7\bar{N}}\ (\epsilon_1+\epsilon_2)\eta_MX^T\ 0\ (\epsilon_1+\epsilon_2)X^T\bar{\lambda}_1^T\ 0\end{bmatrix}^T, \\
\varPhi_{12} &= \begin{bmatrix}0\ 0\ 0\ 0\ \epsilon_1\mu^2F\ 0_{\bar{N}\times7\bar{N}}\end{bmatrix}^T, \\
\varPhi_{13} &= \begin{bmatrix}0\ 0\ 0\ 0\ 0\ \epsilon_2\mu^2F\ 0_{\bar{N}\times6\bar{N}}\end{bmatrix}^T, \\
\bar{\mathcal{F}} &= \begin{bmatrix}c_1(\mathcal{A}\otimes\mathcal{M}_1)F\ c_2(\mathcal{B}\otimes\mathcal{M}_2)F\ 0\ 0\ X\ X\ I\ \bar{\delta}\end{bmatrix}.
\end{aligned}$$

Furthermore, if the LMI is feasible, the desired controller gain matrix are given by $K_i = X_iY_i^{-1}, O_i = F_i^{-1}\bar{O}_iF_i^{-1}$, $i = 1, 2, ..., N$.

proof 2 *Based on the Schur complement lemma,* $\varSigma$ *in* (18) *can be rewritten as the following form.*

$$\varSigma = \begin{bmatrix} \hat{\varSigma}_{11} & \hat{\varSigma}_{12} & \hat{\varSigma}_{13} & \varSigma_{14} & \varSigma_{15} \\ * & -PR^{-1}P & 0 & 0 & 0 \\ * & * & -PR^{-1}P & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix}, \tag{29}$$

where

$$\hat{\varSigma}_{11}=\begin{bmatrix}\hat{\varXi}_{11} & \varXi_{12} & \varXi_{13} & 0 & \varXi_{15} & \varXi_{16} & P & P\bar{\delta}\\ * & -T & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \varXi_{33} & 0 & 0 & 0 & 0 & 0\\ * & * & * & \varXi_{44} & R & 0 & 0 & 0\\ * & * & * & * & \varXi_{55} & 0 & 0 & 0\\ * & * & * & * & * & -O & 0 & 0\\ * & * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & * & -I\end{bmatrix},$$

$$\begin{aligned}\hat{\varXi}_{11} &= c_1P(\mathcal{A}\otimes\mathcal{M}_1)+c_1(\mathcal{A}\otimes\mathcal{M}_1)^TP+Q-R+T,\\ \hat{\varSigma}_{12} &= \eta_M(P\mathcal{F})^T,\\ \hat{\varSigma}_{13} &= \begin{bmatrix}0\,0\,0\,0\,0\,0\,0\,\eta_MP\mathcal{W}\end{bmatrix}^T,\\ \varSigma_{15} &= \begin{bmatrix}\bar{\lambda}_2\,0\,0\,0\,0\,0\,0\,0\end{bmatrix}^T.\end{aligned}$$

Since $-PR^{-1}P\le -2P+R$ *[22], replacing* $-PR^{-1}P$ *by* $-2P+R$*, then we can get* $\varSigma\le\hat{\varSigma}$*, where*

$$\hat{\varSigma}=\begin{bmatrix}\hat{\varSigma}_{11} & \hat{\varSigma}_{12} & \hat{\varSigma}_{13} & \varSigma_{14} & \varSigma_{15}\\ * & -2P+R & 0 & 0 & 0\\ * & * & -2P+R & 0 & 0\\ * & * & * & -I & 0\\ * & * & * & * & -I\end{bmatrix}, \tag{30}$$

The matric $\hat{\varSigma}$ *can be rewritten as*

$$\hat{\varSigma}=\hat{\varSigma}_1+Z_1^T\hat{\triangle}B+B^T\hat{\triangle}Z_1+Z_2^T\hat{\triangle}B+B^T\hat{\triangle}Z_2, \tag{31}$$

where

$$\begin{aligned}\hat{\varSigma}_1 &= \hat{\varSigma}|_{\hat{\triangle}=I},\hat{\triangle}=diag_N\{\triangle\},\\ Z_1 &= \begin{bmatrix}0\,0\,0\,0\,I\,0_{\bar{N}\times7\bar{N}}\end{bmatrix},\\ Z_2 &= \begin{bmatrix}0\,0\,0\,0\,0\,I\,0_{\bar{N}\times6\bar{N}}\end{bmatrix},\\ B &= \begin{bmatrix}K^TP\,0_{\bar{N}\times7\bar{N}}\;\eta_MK^TP\;0\;K^T\bar{\lambda}_1\;0\end{bmatrix}.\end{aligned}$$

By using the Lemma 2, for any $\epsilon_1,\epsilon_2>0$*, it can be obtained that*

$$\hat{\varSigma}\le\hat{\varSigma}_1+(\epsilon_1+\epsilon_2)B^TB+\epsilon_1^{-1}Z_1^T\hat{\triangle}^2Z_1+\epsilon_2^{-1}Z_2^T\hat{\triangle}^2Z_2. \tag{32}$$

Since $\triangle=diag_n\{\triangle_j\}$ *, and* $|\triangle_j|\le\mu_j$*, we can rewrite the inequality as*

$$\hat{\triangle}^2\le\mu^2I, \tag{33}$$

where $\mu=max\{\mu_j\},j=1,2,...,n$*.*

Then, the following inequality can be derived from the combination of (32) *and* (33)*,*

$$\hat{\varSigma}\le\hat{\varSigma}_1+(\epsilon_1+\epsilon_2)B^TB+\epsilon_1^{-1}Z_1^T\mu^2Z_1+\epsilon_2^{-1}Z_2^T\mu^2Z_2. \tag{34}$$

By using the Schur complement lemma, (34) *is equivalent to the following matrix inequality*

$$\hat{\varSigma}\le\begin{bmatrix}\hat{\varSigma}_1 & (\epsilon_1+\epsilon_2)B^T & \epsilon_1\mu^2Z_1^T & \epsilon_2\mu^2Z_2^T\\ * & -(\epsilon_1+\epsilon_2)I & 0 & 0\\ * & * & -\epsilon_1\mu^2I & 0\\ * & * & * & -\epsilon_2\mu^2I\end{bmatrix}. \tag{35}$$

Letting $P = diag_N\{P_i\}$, and defining $P^{-1} = F, P_i^{-1} = F_i$, pre- and post-multiplying both sides of (35) with $\Omega = diag\{S, F, I, I\}$ and Ω^T respectively, where $S = diag\{F, F, F, F, F, F, I, I, F, F, I, I\}$, and the new matrices can be defined as $KF = X, FQF = \bar{Q}, FRF = \bar{R}, FTF = \bar{T}, FOF = \bar{O}$ one can obtain

$$\Pi = \begin{bmatrix} \bar{\Sigma}_1 & \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & -(\epsilon_1+\epsilon_2)FF & 0 & 0 \\ * & * & -\epsilon_1\mu^2 I & 0 \\ * & * & * & -\epsilon_2\mu^2 I \end{bmatrix}, \tag{36}$$

Replacing $-FF$ by $-2F + I$, we know that $\Pi \le \Psi$, which means $\Sigma \le \Pi \le \Psi \le 0$. This completes the proof.

Remark 9 *By solving the LMI, we can obtain that $K = XF^{-1}$ and $O = F^{-1}OF^{-1}$, that is, $K_i = X_iF_i^{-1}, O_i = F_i^{-1}O_iF_i^{-1}$.*

Remark 10 *Since $\bar{\triangle}$ is a bounded matrix, it is not easy to deal with it when solving the LMI. So we use Schur complement and Lemma 2 to deal with $\bar{\triangle}$ in this paper to make it more feasible to solve the LMI.*

4 Numerical Examples

In this section, a numerical example is used to verify the validity of the criteria established in this paper.

A drive-response complex network (1) and (2) with $N = 5$ nodes are considered. Moveover, a coupled network is given in Fig. 3.

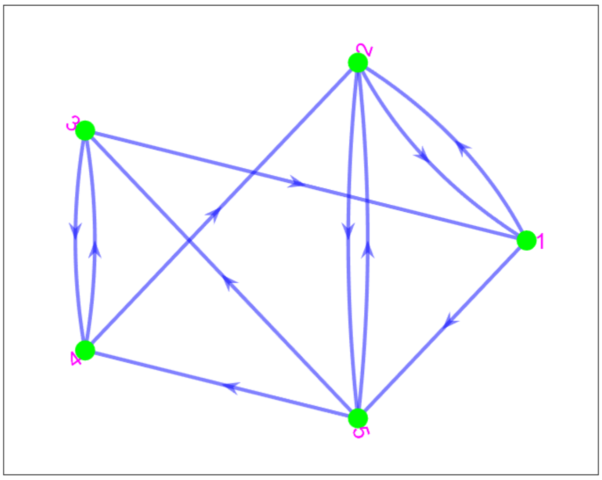

Figure 3. A coupled network with 5 nodes.

The connection matrix is given to be

$$L = \begin{bmatrix} 0\,1\,0\,0\,1 \\ 1\,0\,0\,0\,1 \\ 1\,0\,0\,1\,0 \\ 0\,1\,1\,0\,0 \\ 0\,1\,1\,1\,0 \end{bmatrix}.$$

Let $D = \text{diag}[2,2,2,2,3]$ and the weight configuration matrices $\mathcal{A} = \mathcal{B} = \mathcal{C} = L - D$.

For each node i $(i = 1,2,3,4,5)$, we select the following nonlinear function $v(e_i(t))$

$$v(e_i(t)) = \begin{bmatrix} e_{i1}(t)\sin(0.1e_{i1}(t)) \\ 0.1e_{i1}(t)\sin(0.1e_{i2}(t)) \end{bmatrix}.$$

We can calculated the upper bound matrix of v is

$$\lambda_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The cyber-attack signal function is assumed as follows

$$\phi(u_i(t)) = 0.1u_i(t) + \tanh(0.1u_i(t)),$$

and the upper bound matrix of $\phi(u_i(t))$ can be calculated as

$$\lambda_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

Suppose that the coupling strengths $c_1 = c_2 = c_3 = 1$ and the inner coupling matrices

$$\mathcal{M}_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathcal{M}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathcal{M}_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

For given the threshold parameter

$$[\sigma_1, \sigma_2, \ldots, \sigma_5] = [2,2,2,2,2] \times 10^{-1},$$

time delays $\varsigma = 0.1$ and the time-varying delay in the network communication

$$[\eta_1, \eta_2, \ldots, \eta_5] = [5,5,5,5,5] \times 10^{-2}.$$

Let the sampling period

$$[h_1, h_2, \ldots, h_5] = [4,3,4,3,4] \times 10^{-2}.$$

We select the quantization density as $\varrho_j = 1/3$, and the expectation of the probabilities of cyber-attacks $o_1 = o_2 = o_3 = 0.5$, and $\epsilon_1 = \epsilon_2 = 0.01$.

Then, the LMI (27) can be solved and a set of the feasible solutions can be derived as follows:

$$P_1 = \begin{bmatrix} 1.1195 & 0 \\ 0 & 1.1195 \end{bmatrix}, \qquad X_1 = \begin{bmatrix} -1.4877 & 0 \\ 0 & -1.4877 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 1.0351 & 0 \\ 0 & 1.0351 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} -1.5773 & 0 \\ 0 & -1.5773 \end{bmatrix},$$
$$P_3 = \begin{bmatrix} 1.2188 & 0 \\ 0 & 1.2188 \end{bmatrix}, \qquad X_3 = \begin{bmatrix} -1.3783 & 0 \\ 0 & -1.3783 \end{bmatrix},$$
$$P_4 = \begin{bmatrix} 0.7207 & 0 \\ 0 & 0.7207 \end{bmatrix}, \qquad X_4 = \begin{bmatrix} -2.0043 & 0 \\ 0 & -2.0043 \end{bmatrix},$$
$$P_5 = \begin{bmatrix} 0.6496 & 0 \\ 0 & 0.6496 \end{bmatrix}, \qquad X_5 = \begin{bmatrix} -2.7009 & 0 \\ 0 & -2.7009 \end{bmatrix}.$$

Then the parameter of the desired controllers and the triggering matrices are given by

$$K_1 = \begin{bmatrix} -1.6655 & 0 \\ 0 & -1.6655 \end{bmatrix}, \qquad O_1 = \begin{bmatrix} 5.8934 & 0 \\ 0 & 5.8934 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.6327 & 0 \\ 0 & -1.6327 \end{bmatrix}, \qquad O_2 = \begin{bmatrix} 5.3681 & 0 \\ 0 & 5.3681 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -1.6798 & 0 \\ 0 & -1.6798 \end{bmatrix}, \qquad O_3 = \begin{bmatrix} 6.4772 & 0 \\ 0 & 6.4772 \end{bmatrix},$$
$$K_4 = \begin{bmatrix} -1.4446 & 0 \\ 0 & -1.4446 \end{bmatrix}, \qquad O_4 = \begin{bmatrix} 3.4552 & 0 \\ 0 & 3.4552 \end{bmatrix},$$
$$K_5 = \begin{bmatrix} -1.7544 & 0 \\ 0 & -1.7544 \end{bmatrix}, \qquad O_5 = \begin{bmatrix} 3.4527 & 0 \\ 0 & 3.4527 \end{bmatrix}.$$

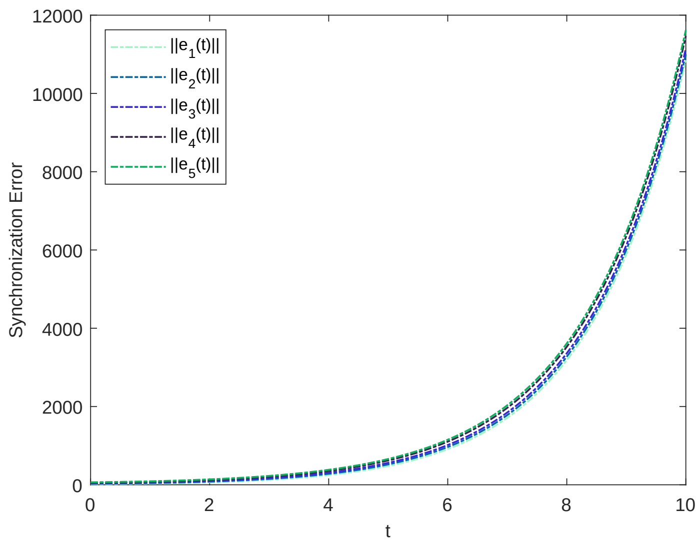

Figure 4. The trajectories of synchronization error $\|e_i(t)\|$ for $i = 1, 2, \ldots, 5$ when the response system is uncontrolled.

Simulation results are shown in Figs. 4-8. From Fig. 4, we can find that Systems (1) and (2) can not achieve synchronization, when the response system (2) is uncontrolled. for any node $i = 1, 2, \ldots, 5$ without control. Then, Fig. 5 shows systems (1) and (2) can achieve synchronization when the response system (2) is controlled with event-triggered scheme. Fig 6 shows the trajectories of control input with quantization and Fig 7 shows the trajectories of control input with cyber-attacks. The release instants and release intervals (RIRIs) are illustrated in Fig. 8.

5 Conclusion and Future Work

In this paper, the event-triggered asymptotic synchronization control problem of DRCNs with and quantization and cyber-attacks has been studied. In order to reduce the communication load and save the network bandwidth in NCSs, sample-data-based event-triggering mechanism have been investigated. Firstly, a coupled error system with delays has been built to describe the performance of the event-triggered mechanism with periodic sampling and the effect of the transmission delay on the NCSs. Then, some sufficient conditions have been derived to guarantee the asymptotic synchronization and the event-triggered

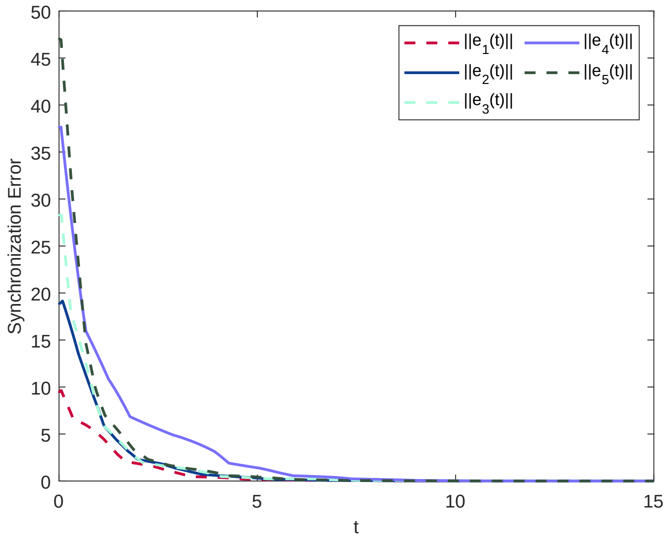

Figure 5. The trajectories of synchronization error $\|e_i(t)\|$ for $i = 1, 2, \ldots, 5$ when the response system is controlled.

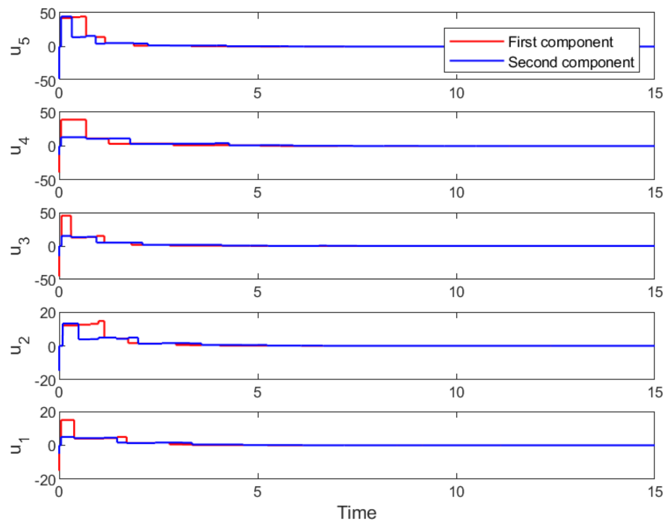

Figure 6. The trajectories of control input $u_i(t)$ for $i = 1, 2, \ldots, 5$ with quantization in (11).

synchronization controllers have been designed. Finally, a numerical simulation example has been given to show the effectiveness of our event-triggering scheme. Recently, stochastic systems have become an interesting research topic (see [44,45]), we will generalize our results to the corresponding stochastic systems in the future.

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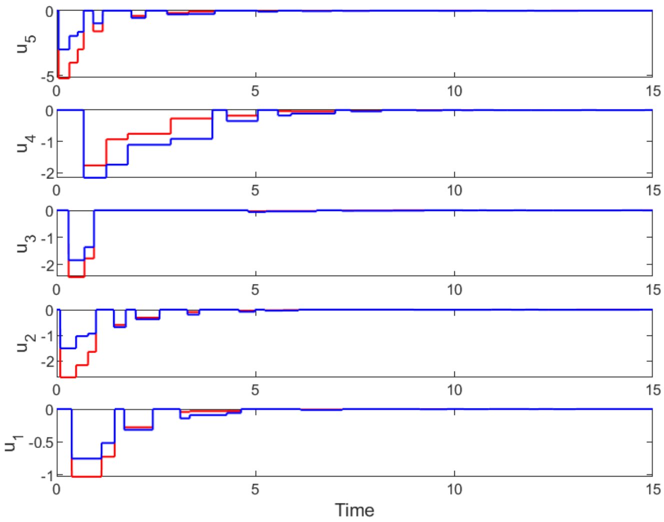

Figure 7. The trajectories of control input $u_i(t)$ for $i = 1, 2, \ldots, 5$ under cyber-attacks in (12).

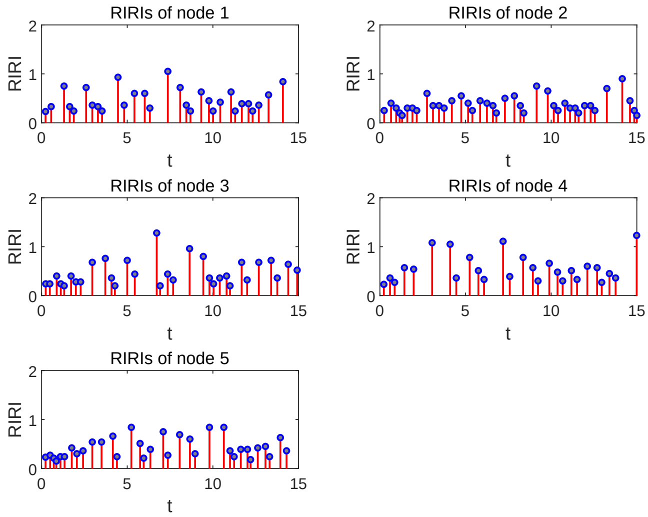

Figure 8. The event-triggered instants and intervals.

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