

Holographic Dark Energy Cosmological Model in Scalar Tensor Theory of Gravitation

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Abstract. An attempt has been taken to study the minimally interacting fields, dark matter and holographic dark energy in scalar tensor theory formulated by Saez and Ballester. The exact solution of the field equations is obtained by using the fact that the scalar expansion of the universe is proportional to the shear scalar. Some physical and geometrical aspects of the model are also discussed.

Keywords: Bianchi type VI₀ universe, scalar- tensor theory, holographic dark energy.

1 Introduction

Various observational data of modern cosmology based on measurement indicate that our universe is in the process of an accelerated expansion. This was observed by Reiss et al.[1], SNeIa (Perlmutter et al.[2]), WMAP (Bennett[3]), SDSS(Tegmark[4]) and X-ray (Allen[5]). The concept of dark energy refers to a kind of exotic energy with negative pressure whose origin still remains a mystery. Astronomical observation i.e. Wilkinson, Microwave Astrophysics Probe indicates that the dark energy occupies 76% of the energy of our universe. 20% dark matter and baryonic matter and radiations occupy about 4% of total energy of the universe. The nature of the dark energy and dark matter is unknown and many radically different models have been proposed such as at tiny positive cosmological constant, quintessence (Caldwell; et al [6], Liddle and Scherrer [7], Steinhardt et al. [8], Dvali et al.[9], Deffayet [10]); the nonlinear E(R) models (Capozziello et al. [11], Carroll et al. [12], Nojiri and Odintsov [13]) and dark energy in brine worlds.

Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. So in recent years, there has been a lot of interest in several alternative theories of gravitation. The most important among them are scalar tensor theories of gravitation formulated by Brans and Dicke ([14], Nordtvedt [15] and Saez and Ballester [16]). All versions of the scalar- tensor theories are based on the introduction of a scalar field φ into the formulation of general relativity, this scalar field together with the metric tensor field then forms a scalar tensor field representing the gravitational field.

In Saez-Ballester theory, the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields and suggests a possible way to solve missing matter problem in non-flat FRW cosmologies.

The Saez-Ballester [16] field equations are

$$G_{ij} - \omega\varphi^n \left(\varphi_{,i}\varphi_{,j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k} \right) = -(T_{ij} + \bar{T}_{ij}), \quad (1)$$

The scalar field φ satisfies the following equation

$$2\varphi^n \varphi^i_{,i} + n\varphi^{n-1} \varphi_{,k}\varphi^{,k} = 0 \quad (2)$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, T_{ij} and \bar{T}_{ij} are energy momentum tensor of matter and holographic dark energy, ω and n are constants, comma (,) and semicolon (;) denotes partial and covariant differentiation respectively.

A detailed discussion of Saez-Ballester cosmological models is contained in the work of Saez[16], Singh and Agrawal[17], Shri Ram and Tiwari[18], Recently Adhav et al. [19] have studied Axially symmetric non-static domain walls in scalar-tensor theories formulated by Brans and Dick [14] and Saez-Ballester[16].

Using holographic principal of quantum gravity theory (Susskind [20]) a viable holographic dark energy model was constructed by Li [21]. The holographic dark energy model is successful in explaining the observational data. The several authors, in particular, Sarkar and Mahanta [22] studied the evolution of holographic dark energy in Bianchi type -I space time with constant deceleration parameter. Recently Sarkar [23] studied holographic dark energy model in Bianchi type -I space time with linearly varying deceleration parameter and established a corresponding with generalized Chaplignin gas models of the universe. Setare [24] studied holographic dark energy model in Brans-Dicke theory. Sarkar [25, 26, 27] has investigated Bianchi type-II cosmological model with interacting holographic dark energy. Very recently Setare and Vanegas [28] have investigated the cosmological dynamics of interacting holographic dark energy model. Kiran et al. [29] have studied Bianchi type- V minimally interacting holographic dark energy model in the scalar- tensor theory of gravitation proposed by Saez and Ballester. Adhav et al. [30] have investigated plane symmetric space- time with interacting dark matter and holographic dark energy exponential volumetric expansion. Reddy at al. [31] have discussed Kaluza-Klein minimally interacting holographic dark energy model in scalar tensor theory of gravitation. Recently Rao, V.U.M.et al. [32] have constructed Kantowski-Sachs holographic dark energy in Brans-Dicke theory of gravitation. Motivated with these works, we have studied in this paper, minimally interacting fields dark matter and holographic dark energy in Saez and Ballester scalar tensor theory of gravitation. Our paper is organized as follows.

In section 2, we derive the field equations. In section 3, we deal with the solution of the field equations in presence of holographic dark energy and dark matter. Section 4 is mainly concerned with the physical and kinematical properties. The last section is the conclusion.

2 Metric and Field Equations

We consider the Bianchi Type VI₀ metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)\exp(-2qx)dy^2 + C^2(t)\exp(2qx)dz^2 \quad (3)$$

where q is nonzero constant and A, B, C are scale functions of t only.

The energy momentum tensor of matter and holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \quad (4)$$

$$\bar{T}_{ij} = (\rho_\lambda + p_\lambda)u_i u_j + p_\lambda g_{ij} \quad (5)$$

where ρ_m, p_λ are the energy densities of matter and holographic dark energy and p_λ is the pressure of the holographic dark energy.

The Saez-Ballester field equations (1) and (2) for the metric (3) with the help of equations (4)-(5) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{q^2}{A^2} + \frac{\omega}{2}\dot{\phi}^2 = -p_\lambda \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{q^2}{A^2} + \frac{\omega}{2}\dot{\phi}^2 = -p_\lambda \quad (7)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{q^2}{A^2} + \frac{\omega}{2}\dot{\phi}^2 = -p_\lambda \quad (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{q^2}{A^2} - \frac{\omega}{2}\dot{\phi}^2 = \rho_m + \rho_\lambda \quad (9)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (10)$$

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \dot{\phi} + \frac{n}{2} \frac{\dot{\phi}^2}{\phi} = 0 \tag{11}$$

where $(\dot{})$ over the symbols A, B, C denotes ordinary differentiation with respect to t .

The continuity equation of the matter and dark energy is given by

$$\dot{\rho}_m + \rho_m(3H) + \dot{\rho}_\lambda + 3H(1 + w_\lambda)\rho_\lambda \tag{12}$$

We assume that the matter and holographic dark energy interact with each other hence both the components conserve separately, so that

The continuity equation of holographic dark energy is

$$\dot{\rho}_\lambda + 3H(1 + w_\lambda)\rho_\lambda = 0 \tag{13}$$

and the continuity equation of matter is

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{14}$$

where $w_\lambda = \frac{p_\lambda}{\rho_\lambda}$ is the barotropic equation of state parameter for holographic dark energy.

The physical and geometrical parameters for the space -time (3), we have the following forms

The average scale factor

$$R = (ABC)^{\frac{1}{3}} \tag{15}$$

The mean Hubble's parameter

$$H = \frac{\dot{R}}{R} \tag{16}$$

an important observed quantity in cosmology is the deceleration parameter q which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 - \frac{\dot{H}}{H} \tag{17}$$

The average anisotropic parameter Δ is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 \tag{18}$$

where $\Delta H_i = H_i - H$ and $H_i (i = 1, 2, 3)$ represent the directional Hubble parameters, the shear scalar expansion(θ), and the shear scalar (σ) are

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{19}$$

$$\sigma^2 = \frac{1}{2} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} \right) \tag{20}$$

3 Solution to the Field Equations

Equation (10) leads to

$$B = kC \tag{21}$$

where k is integrating constant.

Using the equation (20), the system of field equations (6)-(11) reduces to

$$2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C} \right)^2 + \frac{q^2}{A^2} + \frac{\omega}{2} \phi^n \dot{\phi}^2 = -p_\lambda \tag{22}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{q^2}{A^2} + \frac{\omega}{2} \phi^n \dot{\phi}^2 = -p_\lambda \tag{23}$$

$$2\frac{\dot{A}\dot{C}}{AC} + \left(\frac{\dot{C}}{C}\right)^2 - \frac{q^2}{A^2} - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 = \rho_m + \rho_\lambda \quad (24)$$

$$\frac{\ddot{\varphi}}{\varphi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\frac{\dot{\varphi}}{\varphi} + \frac{n}{2}\frac{\dot{\varphi}^2}{\varphi^2} = 0 \quad (25)$$

The field equations (21)-(24) are the system of four equations with six unknowns $A, C, \rho_m, \rho_\lambda, p_\lambda, \varphi$ hence we need additional constraints to obtained solution of the system. We use the scalar expansion θ that is proportional to the shear scalar σ^2 . So that we have (Collins et al. [33])

$$A = C^n \quad (26)$$

using the equations (21), (22) and (25) we obtain,

$$\frac{\ddot{C}}{C} + (1+n)\left(\frac{\dot{C}}{C}\right)^2 + \frac{2q^2}{(1-n)C^{2n}} = 0 \quad (27)$$

we assume

$$\dot{C} = f(C) \quad (28)$$

$$\ddot{C} = ff' \quad (29)$$

where

$$f' = \frac{df}{dC}$$

equating (26), (27) and (28), we have

$$\frac{df^2}{dC} + 2(1+n)\frac{f^2(C)}{C} = \frac{2q^2}{(n-1)C^{2n-1}} \quad (30)$$

equation (29) yields

$$f^2(C) = \frac{q^2}{2(n-1)}C^{2(1-n)} + \frac{\beta}{C^{2(1+n)}} \quad (31)$$

where β is an integrating constant.

Using the suitable choice of transformation i.e. $C = T, x = X, y = Y, z = Z$, the metric (3) reduces to the form

$$ds^2 = -\left[\frac{q^2}{2(n-1)}T^{2(1-n)} + \frac{\beta}{T^{2(1+n)}}\right]^{-1} dT^2 + T^{2n} dX^2 + (kT)^2 \exp(-2qX)dY^2 \quad (32)$$

The scalar field φ in the model is

$$\varphi^{\frac{n+2}{2}} = N\varphi_0 \sqrt{\frac{q^2}{2(n-1)T^{4n}} + \frac{\beta}{T^{4(1+n)}}} \quad (33)$$

where the constant φ_0 is omitted and

$$N = \frac{-(n+2)}{2(n+1)}$$

The pressure of holographic dark energy p_λ of model (32) is

$$p_\lambda = \frac{-q^2}{2(n-1)T^{2n}} + \frac{\beta(2n+1) + \frac{\omega}{2}\varphi_0^2}{T^{2(n+2)}} \quad (34)$$

using equation (14), we get the energy density ρ_m of dark matter

$$\rho_m = \frac{\rho_0}{T^{n+2}} \quad (35)$$

where ρ_0 is integrating constant and energy density ρ_λ of holographic dark energy is

$$\rho_\lambda = \frac{2q^2}{(n-1)T^{2n}} + \frac{2\beta(n+1) - \frac{\omega}{2}\phi_0^2}{T^{2(n+2)}} - \frac{\rho_0}{T^{n+2}} \tag{36}$$

The equation of state parameter w_λ of holographic dark energy is

$$w_\lambda = \frac{\left(\frac{-q^2}{2(n-1)T^{2n}} + \frac{\beta(2n+1) + \frac{\omega}{2}\phi_0^2}{T^{2(n+2)}} \right)}{\left(\frac{2q^2}{(n-1)T^{2n}} + \frac{2\beta(n+1) - \frac{\omega}{2}\phi_0^2}{T^{2(n+2)}} - \frac{\rho_0}{T^{n+2}} \right)} \tag{37}$$

which shows that w_λ is a function of time.

The coincident $r = \frac{\rho_\lambda}{\rho_m}$ i.e. the ratio of dark matter energy density to the dark energy density is given by

$$r = \frac{\frac{2q^2}{T^{n-2}} + \frac{2\beta(n+1) - \frac{\omega}{2}\phi_0^2}{T^{n+2}} - \rho_0}{\rho_0} \tag{38}$$

The matter density parameter Ω_m and holographic dark energy density parameter Ω_λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_\lambda = \frac{\rho_\lambda}{3H^2} \tag{39}$$

From equations (23), (30), (33) and (39), we obtain the overall density i.e. the sum of density parameter as

$$\Omega_m + \Omega_\lambda = \frac{3}{(n+2)^2} \left[(2n+1) - \frac{q^2}{\left(\frac{q^2}{2n+1} + \frac{\beta}{T^4} \right)} - \frac{\frac{\omega}{2}\phi_0^2}{\left(\frac{q^2 T^4}{2(n-1)} + \beta \right)} \right] \tag{40}$$

Here we observe that the sum of energy density parameter approaches to constant at the late time, so at late time the universe becomes flat.

4 Some Physical and Kinematical Properties of the Model

Some physical and kinematical parameters of cosmological model have the following expression

Spatial volume

$$V = kT^{n+2} \tag{41}$$

Scalar expansion

$$\theta = (n+2) \sqrt{\frac{q^2}{2(n-1)T^{2n}} + \frac{\beta}{T^{2(n+2)}}} \tag{42}$$

Shear scalar

$$\sigma^2 = \frac{(n-1)^2}{3} \left(\frac{q^2}{2(n-1)T^{2n}} + \frac{\beta}{T^{2(n+2)}} \right) \tag{43}$$

Hubble parameter

$$H = \frac{n+2}{3} \sqrt{\frac{q^2}{2(n-1)T^{2n}} + \frac{\beta}{T^{2(n+2)}}} \tag{44}$$

Anisotropic parameter

$$\bar{A} = \frac{4}{3} \quad (45)$$

5 Conclusion

In this paper, we have investigated the Bianchi type-VI₀ cosmological model filled with dark matter and holographic dark energy in scalar-tensor theory of gravitation proposed by Saez and Ballester [16]. The exact solution of the field equations is obtained by using the fact that the scalar expansion of the universe is proportional to the shear scalar. It is observed that the holographic dark energy density decreases with the evolution of the universe and the average density parameter approaches to constant at late time, so the universe becomes flat which is supported to recent observations. We observe that the spatial volume V is zero at $T = 0$. At this epoch, the physical parameters H , θ , σ , ρ_m , ρ_λ and p_λ are all infinite. Therefore the model has a point singularity at $T = 0$. As time increases, the spatial volume increases and the physical parameters decrease and ultimately tend to zero as $T \rightarrow \infty$.

Therefore the ratio $\frac{\sigma}{\theta}$ does not tend to zero as $T \rightarrow \infty$. Therefore the shear scalar does not tend to zero faster than the expansion at late time.

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